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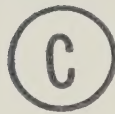
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COMPARISON OF URBAN AND RURAL TURBULENCE STATISTICS
AT EDMONTON, ALBERTA

by



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A THESIS

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ABSTRACT

Investigation of urban effects on the power spectrum of turbulence is of interest in estimating the rate of diffusion of pollution in the city, and also in determining the applicability of theories concerning atmospheric turbulence in a rough, non-homogeneous environment. In this study, turbulence observations from an urban site and a rural site at Edmonton, Alberta, were compared. Periods with light winds and stable lapse rates were chosen, because the rate of diffusion is more likely to be important under these conditions. The observations analyzed in this study were obtained on six days in September, 1970.

The turbulence measurements were made using propellor-and-bivane anemometers (Vector-Vanes) mounted at a height of 2.7 m. Supporting lapse rate measurements were also obtained. The analogue signals from the Vector-Vanes were converted to digital form for analysis. Power spectra were computed by averaging short, modified periodograms. Turbulent intensities were computed and corrected for attenuation at short wavelengths. All turbulence statistics were computed in terms of polar coordinates. These statistics were used as approximations to their Cartesian analogues.

It was found that the turbulent intensity was more than three times greater at the urban site than at the rural site.

The urban wind speed, elevation, and azimuth spectra agreed well with the Kolmogorov $-5/3$ law at wavelengths shorter than 8 to 50 times the distance to the ground. Rural speed spectra agreed with the $-5/3$ law at longer wavelengths than did rural elevation spectra. It was concluded that isotropy was not a necessary condition for a $-5/3$ law to apply to turbulence spectra.

The position of the peak in the rural elevation angle spectrum agreed with similarity theory and recent observations. The position of the peak in the urban elevation spectrum was displaced toward longer wavelengths. Two alternative hypotheses were proposed: either that similarity theory was not obeyed close to the ground in a rough urban environment, or that polar and Cartesian spectra were not similar in shape when the turbulent intensity was high.

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CHAPTER I

INTRODUCTION

Meteorologists and geographers have been interested in the effect of a city on its climate for some time. Studies have been carried out on the urban heat island, urban effects on precipitation, and urban effects on the mean wind field. One aspect of a city's influence on its climate that has only recently received attention is its effect on the magnitude and scales of air turbulence.

A knowledge of the characteristics of turbulence in the city is useful in two major ways: first, in estimating the effects of wind loading on tall buildings (Davenport, 1967); and second, in estimating the rate of diffusion of pollutants. This study is directed toward providing some data applicable to the second of the above problems. An experimental comparison of urban and rural turbulence will be made under conditions when the rate of diffusion of pollutants would be expected to be important to the inhabitants of a city. That is, when diffusion occurs slowly; in stable air with light winds.

The relationship between the statistics of homogeneous, stationary turbulence and the diffusion of particulate matter in such a field was treated theoretically by Taylor in 1921. He related the rate of diffusion to the

intensity of turbulence and the Lagrangian auto-correlation coefficient. Because the auto-covariance and the power spectrum are Fourier transforms of one another, it is sufficient to specify the Lagrangian power spectrum in order to estimate the rate of diffusion. In practice, the rate of diffusion has been measured and used to estimate the Lagrangian power spectrum since the latter is very difficult to obtain by direct means (Lumley and Panofsky, 1964).

Various attempts have been made to relate the Lagrangian power spectrum to the Eulerian power spectrum, which is easier to measure. Among these studies is that of Hay and Pasquill (1957), who suggested that Eulerian and Lagrangian spectra were similar except that the time scales were related by a constant factor. Using this hypothesis, the Eulerian time spectra to be presented in this study may be related to the rate of diffusion of pollutants.

A previous study comparing urban and rural turbulence is that of Bowne, Ball, and Anderson (1968), at Fort Wayne, Indiana. They found that turbulent intensities were greater in the city than in the country and that there were some systematic differences in the spectra. Consequently, there may be some significant differences in the diffusion of pollutants in rural and urban environments. Also, analysis of these differences may shed some light on the range of applicability of similarity theory in predicting turbulence statistics in varying environments.

The intensity and scales of turbulence may be affected by gross meteorological conditions in ways that are not completely understood. In order to isolate the effect of the urban environment, concurrent observations of the three components of the wind were obtained at an urban and a rural site. The observations were taken between 0500 and 0700 Mountain Standard Time, when inversion conditions were likely to prevail. Fast-response propellor-and-bivane instruments ("Vector-Vanes") were mounted on 2.7-m portable masts at both sites. Supporting estimates of lapse rates were made using a tethered balloon and data available from thermometers at various levels on a downtown office building. From these data, urban and rural turbulent intensities and power spectra were calculated and compared. The results to be presented were obtained on six days in September, 1970.

Chapter II contains a detailed description of the collection and recording of the data. Mean wind speed and lapse rate tables which are used in later analyses are presented in chapter III. In chapter IV, the method used to calculate power spectra is described and the urban and rural spectra are compared. Chapter V is concerned with the calculation and comparison of turbulent intensities.

CHAPTER II

COLLECTION OF THE DATA

2.1 The Observing Sites

2.1.1 The Rural Site

The rural site was located on the University of Alberta farm at Ellerslie, about 5 km south of the city limits. Its position is indicated on Figure 1. The instruments were situated near the eastern edge of a field of mown hay about 10 cm high. A band of trees bounded the field on the west. These trees were approximately 15 m high and 400 m from the observing site. A group of farm buildings was about 100 m to the north. Trees also occurred about 200 m to the southeast; and to the northeast. The ground slopes downward to the northeast at less than 1 percent. The only significant topographic feature nearby is the Whitemud Creek gully, which may be seen on the aerial photograph (Figure 2). It runs in a northeasterly direction and lies at a distance of 1 km to the west of the site. The site is felt to be typical of the cultivated parkland of Central Alberta.

2.1.2 The Urban Site

In 1970 the population of the city of Edmonton was 435,000; the built-up area was about 16 km square. The

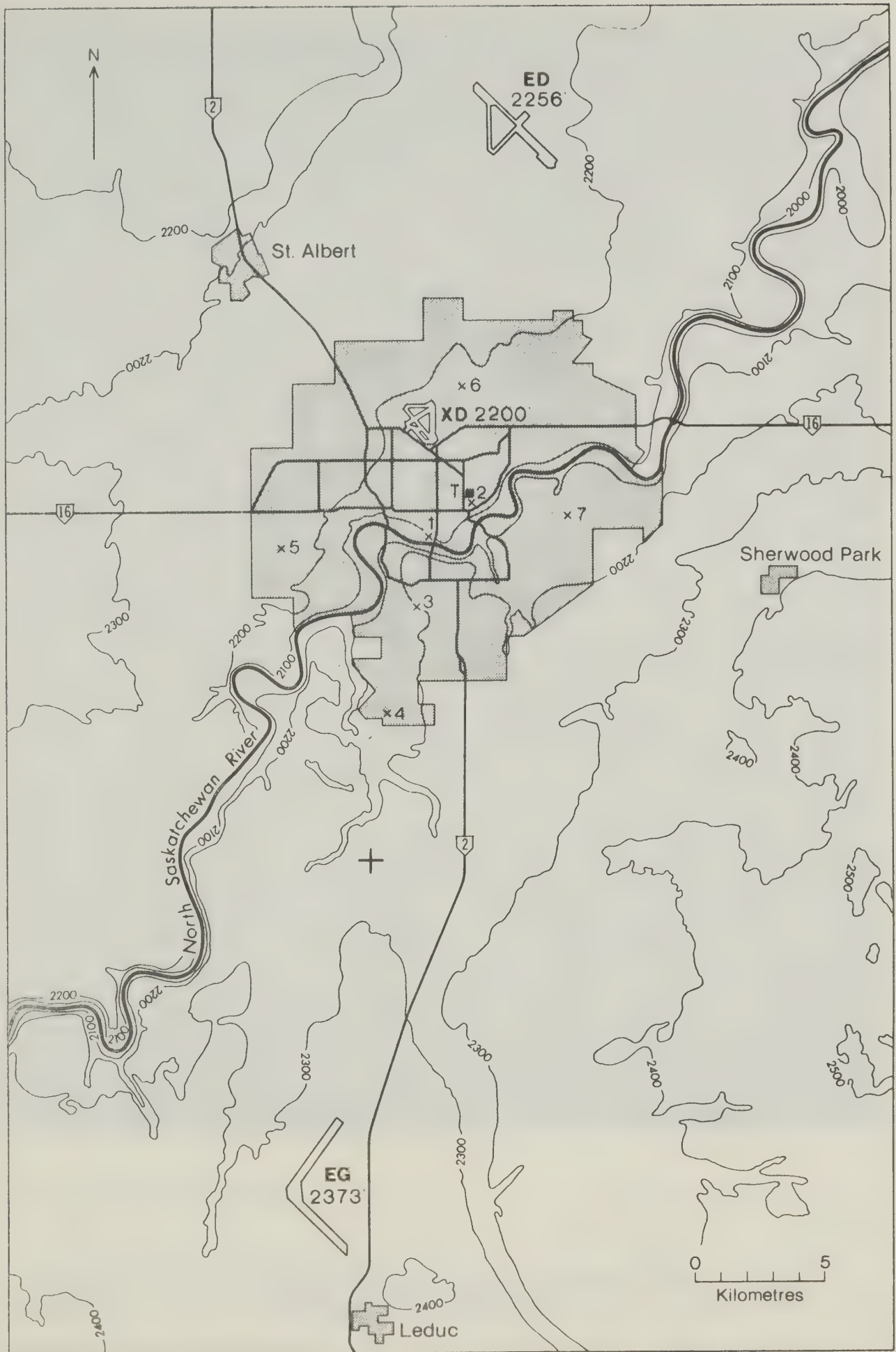
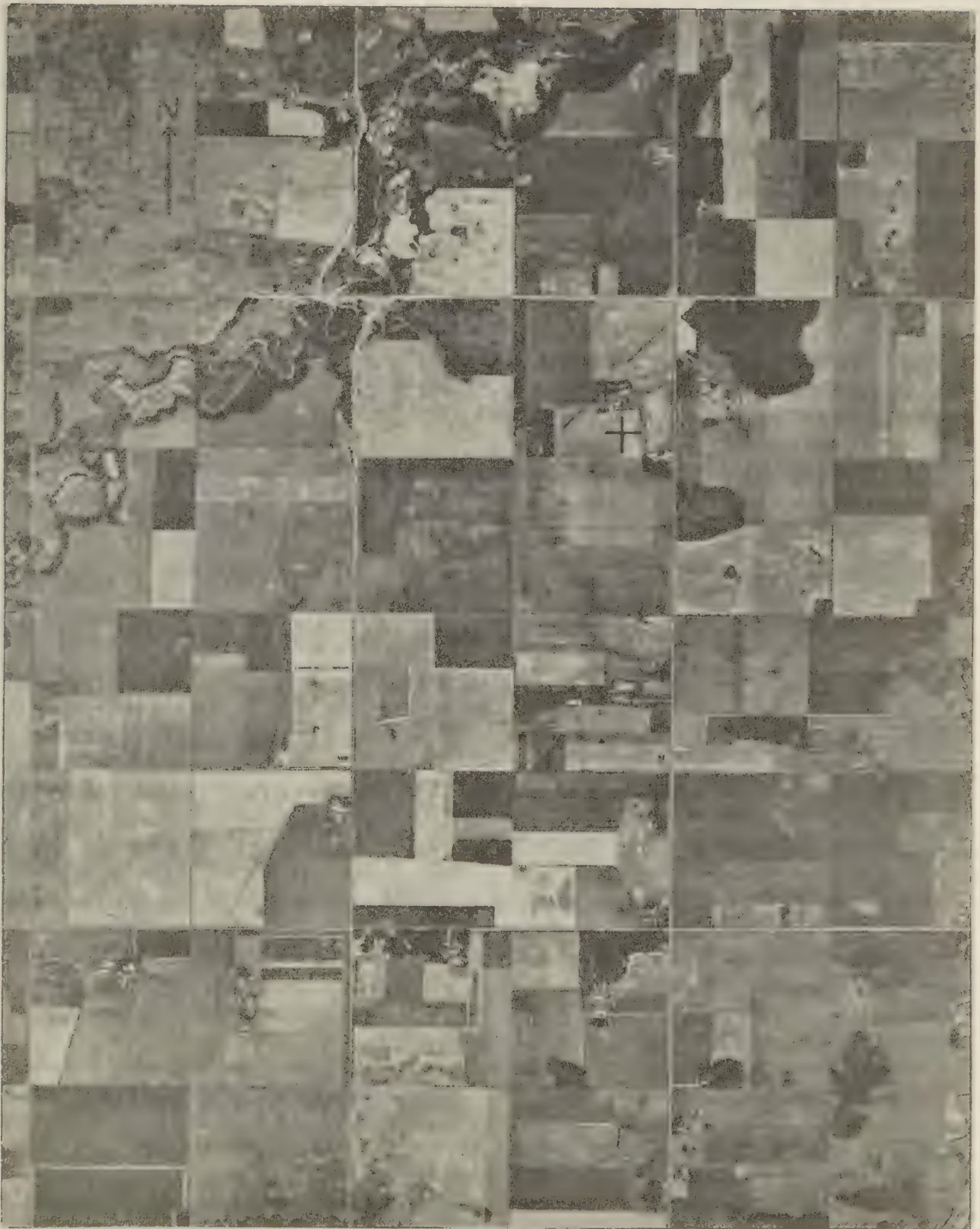


Figure 1 Map of Edmonton and vicinity

T - CN Tower
 2 - Urban site
 + - Rural Site



+ Observing Site

0 ————— 1 km

Figure 2 Aerial Photograph of the Rural Site
(Government of Alberta)

North Saskatchewan River valley, which is 50 m deep, bisects the city from southwest to northeast. In view of the high artificial roughness and inhomogeneity of the city, its topography is not thought to be a significant influence on the low-level turbulence.

The observing site was on a concrete plaza north of 101A Avenue between 100 and 99 Streets. This location was near the geometric centre of the city and on the eastern fringe of the most densely built-up area. The plaza was 30 m by 75 m with the long axis running east-west. The plaza was bounded on the north by the six-story Edmonton Public Library. The instruments were placed 25 m south of the library, approximately centered between two 9-m square one-story buildings in the southwest and southeast corners of the plaza. There was a 10-story building 75 m to the west of the instrument site. The buildings to the south and east were of 1 to 4 stories and their height dropped off gradually to the east. Figure 3 is a sketch map of the urban site.

2.2 Turbulence Measurements

Turbulence measurements were made using two three-dimensional anemometers (Vector-Vanes) manufactured by Meteorology Research, Inc. The anemometers consisted of a bivane and a propeller mounted on a shaft 51 cm long. The wind speed was measured by a light-beam chopper attached to the propeller; elevation and azimuth angles of the vane were

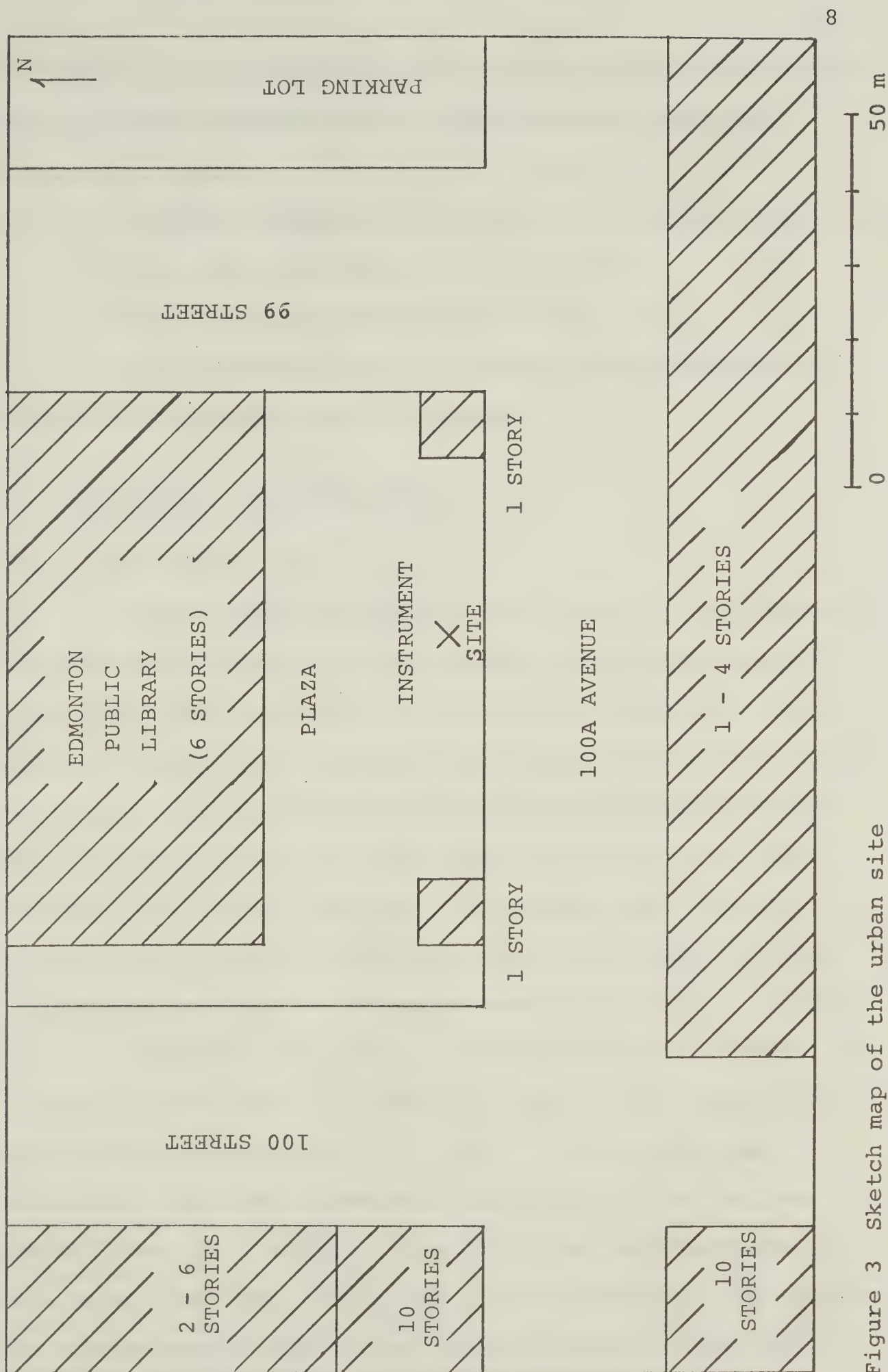


Figure 3 Sketch map of the urban site

measured by potentiometers. The output was in analog form. The pertinent manufacturer's specifications for these instruments were:

Propeller response distance $L = 2 - 3$ feet (0.6-0.9 m)

Vane delay distance $D = 2 - 3$ feet

Vane damping coefficient $\rho = 0.4 - 0.7$

The anemometers were mounted on portable masts so that the vane pivot was 2.7 m high.

2.3 Estimation of Lapse Rates

2.3.1 The Rural Site

Temperature measurements were taken at approximately 30 m intervals from 1 to 150 m above the ground, using a thermistor bead suspended from a tethered balloon. The system consisted of a mercury battery and Wheatstone bridge to measure the resistance of the bead by the null method. The time constant of the bead proved to be too fast for a reliable null to be obtained. The system was therefore recalibrated using the imbalance current at four standard resistance settings to indicate bead temperature. Simple linear regression was used to fit the calibration data; the standard error was 0.1 C and the slope of the calibration curve was approximately $2.1 \text{ C } \mu\text{A}^{-1}$. A pilot-balloon theodolite was used to measure the angle of the balloon tether from the vertical. The height of each measurement was calculated from the length of the tether and this angle. No allowance was made for sag in the tether. This sag

(maximum difference between the chord and the arc) was estimated to be less than 10 percent of the arc length. A 10 percent sag would introduce a systematic underestimation of the absolute value of the lapse rate of approximately 2 percent.

Two flights were made during most runs with measurements taken on both ascent and descent. Measurements at each level were approximately symmetrical in time about the mean time for the ascent. If the temperature at each level changed slowly during a run an unbiased estimate of the mean would be obtained by this technique. In view of the probable error in each measurement of approximately 0.2 C, no attempt was made to measure changes in lapse rate with time or height. All the data for each run were fitted by a straight line using simple linear regression. The standard error of the slope averaged about 10 percent.

2.3.3 The Urban Site

Geoscience Research Associates Ltd., under contract to the Department of Health, Government of Alberta, have taken vertical temperature difference measurements from the CN Tower building at 104 Avenue and 100A Street since 1967. The CN Tower is about 400 m north of the urban observing site. Data from the CN Tower, along with data from other towers across Canada, are published regularly (in the Meteorological Tower Bulletin) by the Atmospheric Environment Service of Canada. The data used here are one-hour averages

of the temperature difference between instruments at the 17 m and 112 m levels. The thermometers are shielded, aspirated resistance bulbs mounted on 2.6-m booms. The accuracy of temperature differences between the sensors is estimated to be ± 0.1 F (Hage and Longley, 1970). The representativeness of the measurements is open to doubt in view of the proximity of the sensors to the building. The degree of horizontal inhomogeneity in the city is also unknown.

2.4 Recording of the Data

2.4.1 Analogue Recording

The analogue signals from the Vector-Vanes representing wind speed, azimuth, and elevation were recorded by Precision Instrument analogue F. M. Tape recorders at $1 \frac{7}{8}$ i.p.s. (inches per second). At the beginning of each run the Vector-Vane output was disconnected from the tape recorder and zero signal was recorded for about 30 sec. Zero output voltage corresponded to 0° azimuth, -60° elevation, and zero speed. The Vector-Vane was then reconnected and a "full scale" signal was recorded for another 30 sec. Full scale output voltage corresponded to 356° azimuth (because the Vector-Vane had a dead spot between 356° and 360°), $+60^\circ$ elevation, and a speed of 20 miles per hour. The full scale voltage of the urban Vector-Vane was set at 1.4 volts and that of the rural Vector-Vane was set at 4.0 volts. On playback, the gain of the tape recorder was

adjusted so that both the urban and the rural signal had a range of 1.4 volts.

At the end of each two-hour run the procedure was repeated. These calibration marks went through the stages of filtering and analogue-to-digital conversion along with the data, and provided the relationship between the digital signals and the original quantities speed, elevation, and azimuth.

2.4.2 Analogue to Digital Conversion

The data were converted from analogue to digital form by a Redcor Model 720 converter. The output of the converter was in increments of 5 millivolts. The sampling rate was 200 sec^{-1} ; four channels were multiplexed (one blank) so that the effective sampling rate for each channel was 50 sec^{-1} and the Nyquist frequency was 25 Hz. Corresponding digital values for different channels were thus not simultaneous; a phase shift linear with frequency was introduced. The phase shift at the Nyquist frequency was 45° between adjacent channels and 90° between non-adjacent channels.

The analogue signal was played back through low-pass filters to avoid aliasing. Adjustable active filters with an attenuation rate of 24 decibels per octave were used. The cutoff of the filters was set at 18 Hz, or about 0.7 times the Nyquist frequency.

The analogue tapes were played back twice through the digitizer; once at $1 \frac{7}{8}$ i.p.s., the same speed at which they were recorded, and once at 30 i.p.s., which compressed the time scale by a factor of 16. For this second digitizing run the effective sampling rate was 3.125 sec^{-1} , the Nyquist frequency was 1.56 Hz, and the effective filter cutoff was 1.125 Hz. The first digitizing run will be referred to as the high-frequency run, and the second as the low-frequency run.

Because of the limited frequency response of the Vector-Vanes at high wavelengths, combined with the low wind speeds encountered during the trials, almost all of the information available from the analogue turbulence signal was contained in the low-frequency digitized data. The high-frequency digitized data proved useful in demonstrating that aliasing had been avoided in the low-frequency data.

CHAPTER III

MEAN WIND SPEEDS AND LAPSE RATES DURING THE TRIALS

Lapse rates during the six trials, estimated by the methods described in Chapter II, are given in Table 1. On September 4 the electrical connection to the tethered balloon at Ellerslie became intermittent due to the strain on the cable. Only two measurements, at 1 m and at 50 m, were made before the breakdown. These measurements indicated a decrease in temperature of 0.1 C from bottom to top of the layer. The uncertainty is very large, so this value is omitted from the table. No data were available from the CN Tower for September 13.

TABLE 1

Lapse Rates during the Trials

Date		Lapse Rate (C/100 m)	
		Rural (Balloon)	Urban (CN Tower)
Sept.	4	..	+1.3
	5	-2.1	-0.2
	9	-3.2	-0.8
	13	-6.7	..
	17	-5.5	-0.8
	20	-2.3	+0.7

It may be seen from the table that an inversion was present during five out of five trials at Ellerslie, excepting September 4. Inversions were present downtown on three days. The lapse rate downtown on September 20 was slightly less than the dry adiabatic, and on September 4 it was slightly greater than the dry adiabatic. The lapse rate downtown was more unstable than that at Ellerslie on the four days when concurrent observations were available.

Mean winds given in Table 2 were obtained from a 10-m cup-and-vane anemometer at Ellerslie and from the Atmospheric Environment Service observations at Edmonton Industrial Airport (XD), which is located in the northwest quadrant of the city. It may be seen that the more unstable urban lapse rates of September 4 and September 20 were associated with higher wind speeds than occurred during the other four trials.

TABLE 2

Mean Wind Speed at 10 m

Date	Rural (Ellerslie)		Urban (XD)	
	Direction (° true)	Speed (m sec ⁻¹)	Direction (° true)	Speed (m sec ⁻¹)
Sept. 4	315	5.4	300	7.1
5	150	2.7	180	4.0
9	200	3.1	190	3.6
13	180	2.2	190	2.2
17	270	3.1	210	3.1
20	270	2.7	280	4.9

On September 4 the wind speed at Ellerslie was 5.4 m sec^{-1} , while on the other five days rural wind speeds were between 2.2 and 3.1 m sec^{-1} . The lapse rate observed downtown on September 4 was super-adiabatic. Since strong winds are generally accompanied by breakdown of the surface inversion layer, it is very likely that the rural lapse rate was much greater on September 4 than during the other trials. This conclusion is supported by the limited data available from the tethered balloon on that date. The conclusion that no inversion was present on September 4 at Ellerslie will be used in chapters IV and V.

The mean wind speed at 2.7 m was calculated from the low-frequency digitized Vector-Vane data. The mean longitudinal component $\bar{u}$ is related to the mean speed $\bar{s}$ by

$$\bar{u} = \bar{s} \left(1 - \frac{\sigma_a^2}{2} - \frac{\sigma_e^2}{2} \right) \quad (3.1)$$

to second order, where σ_a^2 is the azimuth variance and σ_e^2 is the elevation variance, both in radians.¹ When the turbulence is intense (σ_a^2 and σ_e^2 are large), the mean wind speed is an overestimate of the mean longitudinal

¹This relationship may be derived from the velocity transformation equations for polar to Cartesian coordinates (with the x-axis parallel to the mean wind):

component. Table 3 contains values of $\bar{s}$ and $\bar{u}$ calculated from (3.1) and the percentage by which $\bar{s}$ exceeds $\bar{u}$. A significant difference between $\bar{s}$ and $\bar{u}$ occurred in the city.

$$u = s \cos e \cos a$$

$$v = s \cos e \sin a$$

$$w = s \sin e$$

In the first equation, let $s = \bar{s} + s'$ and expand the cosines in Taylor series, retaining only second order terms:

$$u \approx (\bar{s} + s') \left(1 - \frac{e^2}{2}\right) \left(1 - \frac{a^2}{2}\right)$$

and average both sides of the equation, again discarding terms of higher than second order, which yields

$$\bar{u} = \bar{s} \left(1 - \frac{e^2}{2} - \frac{a^2}{2}\right)$$

TABLE 3

$\bar{s}$ and $\bar{u}$ at 2.7 m

Date	Rural			Urban		
	$\bar{s}$ (m sec ⁻¹)	$\bar{u}$ (m sec ⁻¹)	$(\bar{s}-\bar{u})/\bar{s}$ (percent)	$\bar{s}$ (m sec ⁻¹)	$\bar{u}$ (m sec ⁻¹)	$(\bar{s}-\bar{u})/\bar{s}$ (percent)
Sept. 4	4.60	4.50	2.2	4.46	3.31	26
5	2.10	2.10	0	1.61	1.37	15
9	..	..	..	1.07	0.91	15
13	2.06	2.06	0	1.03	0.79	24
17	2.28	2.28	0	..	..	..
20	1.97	1.97	0	4.95	4.06	18

CHAPTER IV

POWER SPECTRUM ANALYSIS

4.1 Methods and Limitations of Analysis

4.1.1 Averaged Modified Periodograms

Power spectra of wind speed, vertical vane angle, and horizontal vane angle were calculated from the low-frequency digitized data using a method described by Welch (1967). The method involves dividing the record into sections and calculating a modified periodogram from each section. The power spectrum is obtained by averaging the periodograms. A short summary of the method is presented below, indicating the choices of such things as data windows, and length and number of sections, which were made in this application. The notation is essentially that used by Welch.

The record was divided into K sections, X_k , $k = 1, \dots, K$. Each section consisted of L consecutive points. In this study K was either 38 or 42, L was 1024, and the sections overlapped by one-half their length.

Each section was multiplied by the Hanning data window:

$$\begin{aligned} W(j) &= \frac{1}{2} (1 - \cos 2\pi j/L), \quad j = 0, \dots, L-1 \\ &= 0 \text{ otherwise} \end{aligned} \tag{4.1}$$

and the discrete Fourier transform was taken:

$$A_k(n) = L^{-1} \sum_{j=0}^{L-1} X_k(j) W(j) \exp(-i 2\pi j n / L)$$

$$k = 1, \dots, K \quad n = 0, \dots, L/2 \quad (4.2)$$

The frequency n is expressed in cycles per unit time, where the unit of time is the length of one section of the record, or 328 sec.

The Fourier transform was performed using a Fast Fourier Transform subroutine available in the University of Alberta program library. The algorithm, which was written by G. Sande, is described in detail by Cochran et al. (1967). The length of the series, L , must be an integral power of 2.

Modified periodograms were calculated from each Fourier transform:

$$I_k(n) = 2 U^{-1} \left| A_k(n) \right|^2 \quad n = 0, \dots, L/2 \quad (4.3)$$

where

$$U = L^{-1} \sum_{j=0}^{L-1} W^2(j). \quad (4.4)$$

Division by U is necessary to correct the variance for the effect of pre-multiplication by the data window. The value of U for the Hanning data window is (approximately) $3/8$. The factor of 2 arises from the definition of spectral

density in terms of positive frequencies only. The power spectrum was obtained by averaging the periodograms:

$$S(n) = K^{-1} \sum_{k=1}^K I_k(n) \quad (4.5)$$

The spectral window corresponding to the Hanning data window is given by

$$Q(n) = U^{-1} \left| L^{-1} \sum_{j=0}^{L-1} W(j) \exp(-i 2\pi j n / L) \right|^2 \quad (4.6)$$

When the area under the curve is normalized to unity, the half-width of this spectral window is 0.8 and the height of the first side lobe is 6×10^{-4} times that of the main lobe.

A measure of the stability of the estimates is

$$\frac{\text{Var } [S(n)]}{E [S(n)]} = K^{-1} \left[1 + 2 \sum_{j=1}^{K-1} \frac{(K-j) \rho(j)}{K} \right] \quad (4.7)$$

where

$$\rho(j) = \left[\sum_{k=0}^{L-1} W(k) W\{k+j(L-M)\} \right]^2 \left[\sum_{k=0}^{L-1} W^2(k) \right]^{-2} \quad (4.8)$$

and M is the amount of overlap (in this case $M = L/2$).

For $n = 0$ and $n = L/2$ the variance is greater by a factor of 2. The quantity $\rho(j)$ expresses the degree of dependence between the sections of the record when the spectrum is flat.

For this application $\rho(1) \approx 1/36$ and $\rho(j) = 0$ for $j = 2, \dots, K-1$. Therefore

$$\frac{\text{Var } [S(n)]}{E^2 [S(n)]} = \frac{19}{18} K^{-1} \quad (4.9)$$

For the same length of record, K is almost twice as large when the sections overlap by one-half their length as when they do not overlap. The resolution is unaffected by overlapping, so this technique uses the available data more efficiently. Greater stability is achieved without loss of resolution.

Because for the low-frequency runs K was 38 or greater, the standard deviation of each estimate is about 14 percent of its expected value.

Power spectra were also calculated from the data digitized at 50 sec^{-1} . The method was essentially similar to that given above, except that the sections were 2048 points (41 sec) long and they did not overlap. A noise peak was found at 10 Hz, which may have been due to mechanical noise in the tape recorder. Because the noise peak was almost a decade above the filter frequency for the low frequency runs, it was attenuated by over 70 decibels. This amount of attenuation removes any possibility of aliasing on the low frequency runs. Aside from this demonstration the high-frequency spectra presented little extra information. The response of the Vector-Vanes did

not extend to frequencies much above 1 Hz, and the resolution was insufficient to indicate the shape of the spectra adequately at low frequencies.

4.1.2 Response of the Vector-Vanes at Short Wavelengths

At short wavelengths the power spectrum is attenuated by the response of the Vector-Vanes. Under favourable conditions the observed spectrum may be corrected for this attenuation, extending the useful frequency range of the analysis.

The bivane which was used to measure the azimuth and elevation angles of the wind behaves as a second order system and the propeller behaves as a first order system (MacCready and Jex, 1964). The relevant energy transfer functions are

$$T_1(n) = \left[1 + (2\pi L_o/L)^2 \right]^{-1} \quad (4.10)$$

for the first order system, where L is the wavelength corresponding to n and L_o is the response distance of the propeller, and

$$T_2(n) = \left[(1 - (\omega_n/L)^2)^2 + 4\rho^2 (L_n/L)^2 \right]^{-1} \quad (4.11)$$

for the second order system, where L_n is the natural wavelength of the vane and ρ is the damping coefficient. The natural wavelength is related to the delay distance by

(MacCready and Jex, 1964)

$$L_n = (6.0 - 2.4\rho)D. \quad (4.12)$$

The transfer functions are plotted in Figure 4.

The observed power spectrum $S(n)$ is related to the "true" power spectrum $S_o(n)$ by

$$S(n) = T(n) S_o(n) \quad (4.13)$$

At a wavelength of 4 m, $T_1(n)$ varies from 0.50 for $L_o = 0.6$ m to 0.39 for $L_o = 0.8$ m. $T_2(n)$ varies from 1.56 (overshoot) for $D = 0.8$ m and $\rho = 0.4$ to 0.52 for $D = 0.8$ m and $\rho = 0.7$. The functions then grow rapidly smaller. At a wavelength of 2 m both transfer functions are of the order of 10^{-1} ; at wavelengths equal to the delay or response distance, $T_1(n)$ is about 2.5×10^{-2} and $T_2(n)$ is an order of magnitude smaller.

The uncertainty in the transfer functions resulting from an inexact knowledge of the response and delay distances and the damping coefficient is seen to be large. For this reason, the spectrum at wavelengths shorter than 4 m was discarded. Wind tunnel tests to determine the characteristics of the particular vanes in use would be necessary before correction could be expected to be reliable.

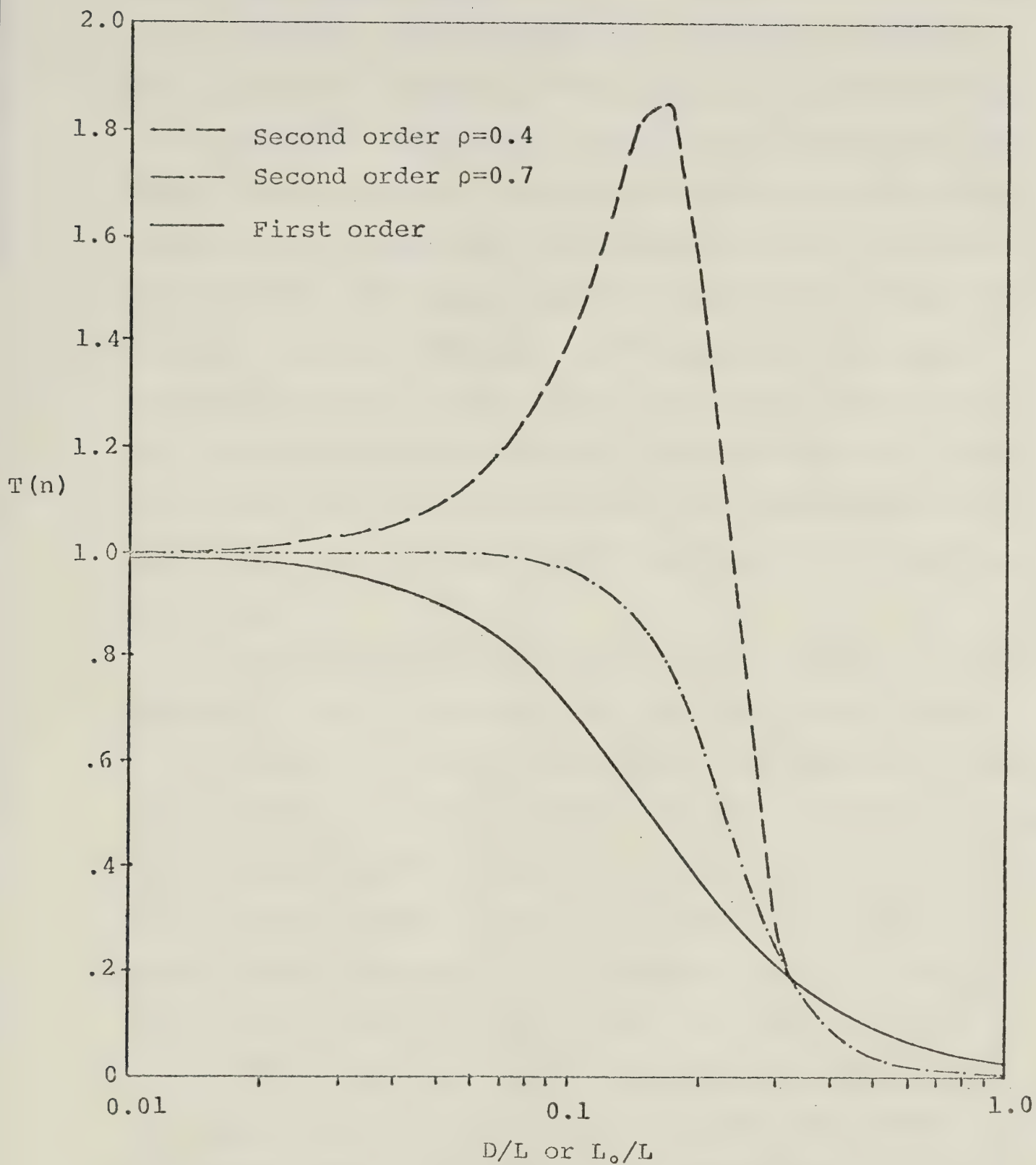


Figure 4 Vector-Vane transfer functions

4.1.3 Quantization Error

When an analogue signal is converted to digital form, noise is introduced. This noise is a result of the rounding off of the analogue signal to the nearest digital value. When the total variance is expressed in the units of the digital scale, the contribution of this quantization noise is $1/12$ unit (Bendat and Piersoll, 1966, p. 281). The smallest variances measured (for horizontal vane angles at Ellerslie) were of the order of 10 digitization units; most variances were at least one order of magnitude larger than this. Under all circumstances the quantization error added less than 1 percent to the variance. It is therefore neglected.

The effect of quantization error on measured power spectra is also of interest. If the quantization errors for successive points are independent, the spectrum of the quantization error will be flat. For the low-frequency runs the Nyquist frequency is 512, considering the time unit to be the length of one record, or 328 sec. Therefore the power spectral density, in digitization units, of the quantization error is $1/12 \cdot 1/512$, or 1.62×10^{-4} .

The logarithmic spectra contained in the Appendix are expressed in these arbitrary units for comparison. It may be seen that at the highest non-dimensional frequency plotted (about 0.65, or a wavelength of 4 meters), $S(n)$ was of order 10^{-2} for most spectra; on September 13 and 17 at Ellerslie $S(n)$ for horizontal vane angle was of order 10^{-3} .

Therefore quantization error contributed insignificantly to $S(n)$ except for the above mentioned horizontal spectra, which contained about 10 percent noise at high frequencies.

At non-dimensional frequencies higher than 0.65, the measured power spectrum decreased rapidly with frequency because of attenuation by the Vector-Vanes. At $f = 1$, most spectral densities were of the order of 5×10^{-4} or less. About 30 percent of this spectral density may be attributed to quantization error. Correction of spectra for the Vector-Vane attenuation, as discussed in the last section, would result in amplification of the quantization error as well and the power spectrum would be seriously overestimated. Reliable correction would require amplification of the signal before digitization to reduce the relative contribution of the quantization error.

Other sources of electrical and mechanical noise were evident in the analogue signal. The electrical noise was estimated at 50 millivolts before low-pass filtering, which corresponded to 10 digitization units. If the spectrum of this noise was flat up to the high-frequency limit of the tape recorder, which was about 300 Hz, the contribution of this noise to the total variance would have been 0.5, in digitization units. The spectral density would have been about 10^{-3} . This noise does not contribute significantly to the total variance, and affects the power spectrum only for rural azimuth spectra on September 13 and 17 at high frequencies. Correction of the spectra for

Vector-Vane response would, however, require the reduction of this noise because it would be amplified by a large factor at high frequencies.

Consideration of the quantization error and other sources of noise, in addition to the effect of uncertainty in the transfer coefficients, indicated that the corrections which were made to the spectra were unreliable. Consequently, the corrections were not applied in the final analysis. The rapid increase of the signal-to-noise ratio with frequency also indicated that success would be unlikely in correcting Vector-Vane spectra at wavelengths shorter than 2 m, even if the transfer functions were known precisely.

4.2 Discussion of Results

The power spectrum estimates are presented as functions of a non-dimensional frequency defined by $f = nz/\bar{u}$, where n is frequency, z is the height of the observations, and $\bar{u}$ is the mean longitudinal component of the wind. This definition was chosen because it represents the ratio of height to wavelength at all levels of turbulent intensity to which Taylor's hypothesis applies. Numerous authors (e.g., Panofsky and McCormick, 1954) defined a non-dimensional frequency by $f = nz/V$ where V is the mean horizontal wind speed. These early studies were concerned mostly with the vertical component of turbulence, and estimates of $\bar{u}$ were probably not available. The definition of non-dimensional frequency used in this study has been used by other authors (e.g., Munn, 1966, p. 70).

The quantities V and $\bar{u}$ are related by

$$\bar{u} = V \left(1 - \frac{\sigma_a^2}{2} \right) \quad (4.14)$$

to second order. Consequently when the turbulent intensity is low, the definitions coincide. When the turbulent intensity is high, n_z/V is smaller than the ratio of height to wavelength. For example, when the lateral turbulent intensity is 0.4, n_z/V is 8 percent smaller than $n_z/\bar{u}$. Because atmospheric turbulence spectra extend over many decades the difference between the definitions is usually negligible. It should, however, be kept in mind when making comparisons with the results of other studies.

Graphs of the power spectra are presented in the usual forms, with the logarithm of non-dimensional frequency as the abscissa. The ordinates are the logarithm of power spectral density $S(n)$, and the product of frequency and spectral density $n \cdot S(n)$. The first ordinate is expressed in arbitrary units. The second ordinate is normalized by $\bar{u}^{-2}$ so that the area under the curve is equal to the turbulent intensity.

4.2.1 Vertical Vane Angle Spectra

The power spectrum of vertical vane angle (elevation) may be identified with the power spectrum of vertical velocity through the approximation

$$w' = \bar{u} e' \quad (4.15)$$

where w' is a fluctuation in vertical velocity and e' is a fluctuation in elevation. Thus the shape of the spectrum of vertical vane angle is approximately the same as that of the vertical velocity. The higher order terms which have been neglected in (4.15) are very complex, and it is not clear under what circumstances the spectra of vertical vane angle and vertical velocity may be of significantly different shape.

The graphs of $n \cdot S(n)$ versus $\log f$ (Figures A-1 to A-6) were smoothed by eye and estimates were made of f_m , the non-dimensional frequency for which $n \cdot S(n)$ is a maximum. The values of f_m which were obtained are given in Table 4.

TABLE 4

f_m for Vertical Vane Angle Spectra

Date	Rural	Urban
Sept. 4	0.2	0.16
5	0.4	0.08
9	..	0.06
13	0.3	0.10
17	0.4	..
20	0.3	0.12
average	0.3	0.10

It was originally proposed by Gurvich (see, e.g., Lumley and Panofsky, 1964) that according to similarity theory,

$$n \cdot S_w(n) = F(nz/V, Ri) \quad (4.16)$$

where $S_w(n)$ is the power spectral density of vertical velocity and F is a universal function of non-dimensional frequency and Richardson number. Therefore f_m should be a universal function of Ri only. Busch and Panofsky (1968) found that f_m was 0.32 under unstable and near-neutral conditions and increased rapidly with increasing stability. Their results were based on many power spectra from different locations in the past decade. Stability was measured as the ratio z/L , where L is the Monin-Obukhov stability length (Monin and Obukhov, 1954). This ratio approaches zero near the ground, and is zero at all heights in neutral air. Consequently f_m should vary little with lapse rate close enough to the ground, and the value of f_m under these conditions should be the value for near-neutral air. Smoothed and averaged spectra for Hanford, Washington, at 3 and 6.1 m show little variation with stability (Busch and Panofsky, 1968). The average value of f_m at Ellerslie (0.3) agrees well with the results of Busch and Panofsky for near-neutral air. The Hanford results indicate that f_m at 2.7 m should have the near-neutral value. Consequently, it is concluded that the value of f_m obtained at Ellerslie is in agreement with the similarity theory.

The average value of f_m at the urban site was 0.1, under conditions when the lapse rate between 50 and 150 m ranged from unstable to stable. This value is not consistent with the universal function proposed by Busch and Panofsky (1968), by which f_m would never be less than 0.32 under any stability. It is therefore suggested that, in the rough urban environment, vertical velocity spectra may not obey similarity theory. This conclusion is not surprising since two assumptions made in the similarity theory are that the height is large in comparison to the roughness height, and that the terrain is homogeneous (Monin and Obukhov, 1954).

If the earlier definition of $f = nz/V$ were used, values of f_m obtained at Ellerslie would be unchanged, since the turbulent intensity was always low. Values of f_m obtained at the urban site would be lower, since $\bar{u}$ was estimated to be an average of 14 percent less than V downtown. The average f_m found using this definition was 0.086. The conclusion that the observed values of f_m downtown are inconsistent with similarity theory is therefore strengthened rather than weakened by the choice of the definition of f .

The possibility also exists that the relationship between the spectra of vertical velocity and vertical vane angle broke down. The relationship appeared valid for the Ellerslie spectra, but turbulent intensities downtown were much higher and the terrain was inhomogeneous. Regretfully this question must be left open for the present.

Bowne, Ball, and Anderson (1968; also Bowne and Ball, 1970), in their comparison of rural and urban turbulence at Fort Wayne, Indiana, concluded that vertical velocity spectra showed some shift of energy to shorter wavelengths in the city. Their measurements were taken at greater heights (about 15 and 60 m) when the rural stability was generally near-neutral. They ascribe the shift toward shorter wavelengths to an increase in mechanical turbulence.

The results of the present study indicate a shift of energy toward longer wavelengths in the city, at a height of 2.7 m. Since the shift occurred on days when inversions were present at both locations, and since the observations were made close to the ground, stability would seem to be a minor factor. If this shift may also be ascribed to mechanical turbulence, the scale of this turbulence is of interest. The horizontal scale of the mechanical turbulence is about $10z$ in Edmonton at 2.7 m. A typical spectrum for Fort Wayne at 53 m (Bowne, Ball and Anderson, 1968, p. 65) indicates maximum energy at $1.25z$. A consequence of similarity theory, which is obeyed reasonably well in the country, is that the scale of turbulence is proportional to height. Combination of the results from this study with those of Bowne, Ball, and Anderson indicates that this is not the case in the city. Rather, it is indicated that the scale is larger near the ground and smaller aloft than in the country; and therefore, increases more slowly with height.

4.2.2 Horizontal Vane Angle Spectra

To first order, a lateral velocity fluctuation v' and an azimuth fluctuation a' are related by:

$$v' = \bar{u} a' \quad (4.17)$$

Therefore, the spectrum of lateral velocity and the azimuth spectrum are of approximately the same shape. The approximation may be expected to be better than the vertical velocity - elevation approximation because wind speed and lateral velocity are not correlated in the same manner as wind speed and vertical velocity (Lumley and Panofsky, 1964, p. 175). Azimuth spectra will therefore be interpreted as lateral velocity spectra (normalized by $\bar{u}^2$).

The time series of horizontal vane angle contained discontinuities where the angle jumped from 0° to 360° . Sections of the records containing these discontinuities were discarded for analysis. During all trials except September 20, the urban horizontal vane angle record contained so many discontinuities that only one or two sections, about 400 sec long, were available for analysis. Consequently, only the low-resolution analysis made from the high-frequency run had adequate stability. This analysis did not extend to low enough frequencies to determine f_m . As a result only one value of f_m is available for the urban site and no conclusive comparison of f_m can be made.

On September 20, f_m was 0.025 downtown and 0.063 at Ellerslie, which indicated a shift of energy toward longer wavelengths in the city (Figure A-13).

Bowne, Ball, and Anderson (1968) found that lateral velocity spectra were roughly similar in shape in urban and rural environments at Fort Wayne. They suggest that there was some tendency toward more energy at shorter wavelengths in the city.

The shifts observed by Bowne, Ball, and Anderson and during the present study are opposite in sense, as was the case for vertical velocity spectra. There is insufficient evidence to conclude that the lateral scale of turbulence behaves similarly to the vertical scale in the city (that is, increasing more slowly with height). The evidence available does raise this possibility, however, and further investigation would be of interest.

An interesting phenomenon took place at Ellerslie during some observations. On September 5, 13, and 17 the azimuth power spectrum tended to increase towards lower frequencies after passing through a minimum in the neighbourhood of $f = 0.01$ (Figures A-11 and A-12). Examination of variances calculated from means over different periods supported this tendency. Table 5 shows the percentage increase in variance that resulted when the means were calculated over 656 sec rather than 328 sec.

TABLE 5

Percentage of Rural Azimuth Variance between
0.00153 Hz and 0.00306 Hz

Date	Percentage
Sept. 4	3
5	32
13	10
17	42
20	26

These values are roughly the amount of variance attributable to frequencies between $1/656$ Hz (1.53×10^{-3} Hz) and $1/328$ Hz (3.06×10^{-3} Hz). These frequencies correspond to values of f of 2×10^{-3} and 4×10^{-3} and wavelengths of 1080 and 540 m respectively, when the wind speed is 2 m sec^{-1} . These estimates were calculated from about 10 sections 656 sec long. The variance between these frequencies may be interpreted as a single spectrum estimate $S(n)$ at some intermediate frequency. The stability of the estimate may be obtained from equation (4.7). Because the sections did not overlap,

$$\frac{\text{Var } [S(n)]}{E^2 [S(n)]} = \frac{1}{10} \quad (4.18)$$

Thus the standard deviation of each estimate is about 30 percent of its expected value. In this light, there appears to be a significant difference between the low-frequency spectrum on September 4 and that during the other four trials. An inversion was present during the four trials when the phenomenon was observed, and the wind speed for the four trials was about 2 m sec^{-1} . It was shown earlier that the wind speed on September 4 was higher than that during the other trials, and that the inversion present on the other days was very likely absent.

It may be concluded that significant power was contained in fluctuations with wavelengths between 540 and 1080 m under stable, light wind conditions.

This phenomenon is most likely the same one that is mentioned by Lumley and Panofsky (1964, p. 178). They speak of "slow-moving features that may have horizontal dimensions of several 100 (sic) meters to a few kilometers", of unknown cause, which have been observed in stable air with light winds.

4.2.3 Wind Speed Spectra

Because fluctuations in the wind speed and in the longitudinal component are equal to first order, the shapes of their spectra should be the same. As in the case of vertical spectra, the conditions under which the first-order assumption breaks down are not clear. Consequently the approximation should be made with caution.

Comparison of urban and rural speed spectra, which are given in the Appendix (Figures A-5 to A-10), does not reveal any obvious difference such as that found for vertical spectra. Values of f_m were obtained from the speed spectra by visual inspection, and are given in Table 6.

TABLE 6

f_m for Speed Spectra

Date	Rural	Urban
Sept. 4	0.01	0.03
5	0.06	0.025
9	..	0.04
13	double peak	0.06
17	no well-defined peak	..
20	0.016	0.025

On September 4 and September 20 there was a small shift of energy toward higher frequencies downtown, but the level of significance was low. On September 5 the shift was in the opposite direction. On September 13 there appeared to be two peaks in the spectrum at Ellerslie. Consequently no systematic differences between rural and urban f_m values are evident.

In the same manner as in the analysis of the horizontal spectra, the difference between variances calculated from 328-sec means and 656-sec means was examined. Again, the difference in variance may be attributed to wavelengths between 540 and 1080 m when $\bar{u}$ is 2 m sec^{-1} . These differences are shown in Table 7.

TABLE 7

Percentage of Speed Variance between
0.00153 Hz and 0.00306 Hz

Date	Rural	Urban
Sept. 4	6	0
5	0.7	1.5
9	..	5.9
13	12	3.7
17	25	..
20	25	0.7

Examination of the table shows that on September 17 and September 20, the behavior of the rural wind speed spectrum was similar to that of the lateral velocity spectrum. That is, a large proportion of the variance was contained in eddies of wavelength between 540 and 1080 m. A similar phenomenon has been observed by other

investigators (Lumley and Panofsky, 1964, p. 184).

The energy between 0.00153 Hz and 0.00306 Hz was low for all urban trials. The absence of the "meandering" observed at the rural site may have two causes. The first is the decrease in stability which was observed in passing from the rural to the urban environment. The second is concerned with the location of the urban site, which was actually between the major roughness elements (the surrounding buildings). The spacing of the buildings may place an upper limit on the wavelength of the disturbances to the flow between them. Further observations using refined techniques for lapse rate measurement would, hopefully, distinguish between these alternatives.

4.2.4 Behaviour of the Spectra at High Frequencies

It was first proposed by Kolmogorov (1941) that, when the Reynolds number is large, there should exist a locally isotropic inertial subrange of the spectrum in which no production or dissipation of energy takes place. The three-dimensional spectrum $E(k)$ is then determined by the wave number k and the total dissipation ϵ only. By dimensional reasoning it follows that

$$E(k) = A \epsilon^{2/3} k^{-5/3} \quad (4.19)$$

where A is a universal constant. Because of the requirement of local isotropy, all three one-dimensional spectra must

also be proportional to $k^{-5/3}$. Also, because $k = fz$, $S(f)$ is proportional to $f^{-5/3}$. The lower limit of the inertial subrange is expected to be at wavelengths short in comparison to the distance to the ground (Lumley and Panofsky, 1964) and short in comparison to the Monin-Obukhov scale length. Since the distance from the ground in the present study is expected to be small compared to the scale length, the first criterion will apply.

Observations have indicated that one-dimensional spectra obey a $-5/3$ power law at wavelengths longer than predicted by the theory (Lumley and Panofsky, 1964). Evidence from the present study supports this conclusion. In fact, the short-wavelength limit of the power spectra in the Appendix is about twice the distance to the ground. Consequently any agreement with the $-5/3$ law that is present is at longer wavelengths than is explained by the theory of the inertial subrange.

Logarithmic plots of spectral density against non-dimensional frequency are shown in the Appendix. The theoretical $-5/3$ law is indicated on each graph by a solid line. It may be seen that the logarithmic spectra generally agreed with a $-5/3$ power law at high frequencies. The fit was best for speed spectra, which exhibited small scatter over a large range of frequency. The rural speed spectrum on September 13, however, showed an unexplained increase in slope above $f = 0.1$ (Figure A-21). The elevation spectra showed more scatter than the speed spectra (Figures A-14 to

A-23). The rural azimuth spectra agreed with a $-5/3$ law on September 5, 17, and 20, but did not on September 4 and 13 (Figures A-24 to A-26).

The urban azimuth spectrum on September 20 (Figure A-26) also fitted a $-5/3$ law well. The high-frequency, low-resolution urban azimuth spectra available for other trials fitted the $-5/3$ law well, but the resolution was insufficient to determine the wavelength at which the law no longer held. These spectra are not presented in the Appendix.

Speed and elevation spectra from the same trial and site are placed together in the Appendix for comparison (Figures A-14 to A-23). It is evident from visual comparison that the $-5/3$ law fitted the rural speed and elevation spectra over a different range. Visual estimates were made of the low-frequency limit of agreement f_L between the spectra and a $-5/3$ power law. These estimates are given in Table 8.

Examination of the table shows that the long-wavelength limit for rural speed spectra ranged between 10 and 50 times the height. The long-wavelength limit for rural elevation spectra took on values between about 3 and 6 times the height. The three rural spectra that agreed with the $-5/3$ law had long-wavelength limits of 8 to 12 times the height.

TABLE 8

Low-frequency Limit f_L of the $-5/3$ Law

Date	Rural		Urban	
	Speed	Elevation	Azimuth	Speed
Sept. 4	.03	.16	*	.02
5	.04	.25	.12	.03
9	..	..	..	.03
13	.02	.23	*	.10
17	.10	.25	.12	..
20	.04	.30	.08	.03
				.02

* These spectra did not fit the $-5/3$ law.

It may be seen that the $-5/3$ law applied to one component when it did not apply to others. The range over which this occurred was from 1 to 4 octaves in the case of rural speed and elevation spectra. Local isotropy cannot occur when all three spectra do not obey the same power law. That is, agreement with a $-5/3$ law was found in the absence of local isotropy. Consequently, isotropy cannot be a necessary condition for a $-5/3$ law, and present theory must be considered incomplete.

CHAPTER V

CALCULATION AND COMPARISON OF TURBULENT INTENSITY

5.1 Methods of Calculation

Turbulent intensity is defined in terms of $\sigma_u/\bar{u}$, $\sigma_v/\bar{u}$, and $\sigma_w/\bar{u}$, where σ_u , σ_v , and σ_w are the standard deviations of the Cartesian velocity components u, v, w . Measures of turbulent intensity are also given by the standard deviation of the elevation angle, σ_e ; the standard deviation of the azimuth angle, σ_a ; and the ratio of the standard deviation of the speed to the mean speed, $\sigma_s/\bar{s}$. These quantities are measured in a spherical polar coordinate system. In this study the Cartesian turbulent intensities will be approximated by their polar analogues. That is, it will be assumed that

$$\begin{aligned}\sigma_a &= \sigma_v/\bar{u}, \quad \sigma_e = \sigma_w/\bar{u}, \quad \text{and} \\ \sigma_s/\bar{s} &= \sigma_u/\bar{u}.\end{aligned}\tag{5.1}$$

These approximations are valid if the corresponding Cartesian and polar velocity components are related by the approximations used in section 4.2.

5.1.1 Choice of the Low-Frequency Limit for Variance Calculations

Variances of azimuth, elevation, and speed were calculated from the low-frequency data. Exceptions were the azimuth variances for the trials on September 4 to 13, which were calculated from the high-frequency digitized data. In the calculation of variance from the low-frequency data, each record was divided into non-overlapping sections 656 sec long. The variance was calculated from each section using the mean calculated over the 656-sec period. The variances obtained from all the 656-sec sections were averaged. The sections were originally chosen to correspond to those used for power spectrum analysis. The length of the power spectrum analysis sections was later cut in half to improve the stability of the spectra.

The calculated variances included energy in the spectrum above a frequency of approximately $1/656$ Hz, or 1.53×10^{-3} Hz. This frequency was chosen to fall in the gap in the spectrum of horizontal wind speed observed by Van der Hoven (1957). His power spectrum covered the range from 2×10^{-7} Hz to 0.25 Hz. A gap in the spectrum was found between 5.6×10^{-5} Hz and 2.8×10^{-3} Hz. A gap in the horizontal speed spectrum between 2.8×10^{-4} Hz and 5.6×10^{-3} Hz was also observed by Walker (1964) under stable and neutral conditions. This gap may be interpreted as the gap between synoptic and micro-scales which must exist for a meaningful definition of atmospheric turbulence.

The graphs of rural and urban power spectra in the Appendix generally show decreasing power toward lower frequencies. This decrease would occur if a spectral gap was being approached. Exceptions to the decreasing trend occur in the rural azimuth and speed spectra which show signs of low-frequency meandering, as discussed in sections 4.2.2 and 4.2.3. These spectra pass through a minimum at a wavelength of about 200 m, and increase again toward lower frequencies. The lowest frequency resolved in the spectral analyses was 3.06×10^{-3} Hz.

On the basis of this observed trend of the power spectra, it may be concluded that most variances calculated in this study include essentially all the energy in the micrometeorological range. The exceptions are the azimuth and elevation variances for trials during which low-frequency meandering occurred. The variance may be underestimated in these cases. The urban azimuth variances for September 4 to 13 must also be excepted. Only short sections of the time series were available for analysis in these cases. Consequently, the estimation of power at low frequencies was not stable. The contribution of low frequencies to these variances was estimated from the only reliable low-frequency urban azimuth spectrum available (September 20).

5.1.2 Correction of Turbulent Intensity for High-frequency Attenuation

By examination of the spectra, it was found that, at wavelengths shorter than 4 m, the power decreased more

rapidly than predicted by the $-5/3$ law. This decrease was due to the limited frequency response of the Vector-Vanes discussed in section 4.1.2. At higher frequencies the spectrum was also attenuated by the low-pass filters. The calculated variances were corrected for this under estimation of the amount of energy at high frequencies by extrapolating the $-5/3$ law to all wavelengths shorter than 4 m. The energy above a frequency n_0 if the $-5/3$ law holds is given by

$$\int_{n_0}^D Cn^{-5/3} dn = 3/2 n_0 S(n_0) \quad (5.2)$$

The value of $S(n)$ at 4 m which was used to make this correction was obtained by fitting a $-5/3$ line to the logarithmic spectra by eye and reading off the value of the ordinate at that wavelength. Because this value was in digitization units, it was multiplied by the calibration factor obtained by the method of section 2.4.1. The energy at wavelengths shorter than 4 m in the unmodified spectrum was removed by subtracting the sum of the power spectrum estimates above this wavelength from the variance, and the term $3/2 n_0 S(n_0)$ was added. Table 9 contains the percentage corrections to the turbulent intensities that were made. To first order, the correction to the turbulent intensity is one-half of the percentage correction to the variance.

TABLE 9

Percentage Corrections to Turbulent Intensities
for High-frequency Attenuation

Date	Rural			Urban		
	Vertical	Horizontal	Speed	Vertical	Horizontal	Speed
Sept. 4	9.8	5.9	1.7	6.2	*	2.4
5	9.6	3.0	1.9	3.4	*	1.4
9	..	..	..	2.9	*	0.7
13	7.7	0.4	1.3	3.8	*	1.8
17	9.4	0.5	1.5	..	..	..
20	6.6	2.0	1.3	6.7	2.6	2.2

* Corrections to these turbulent intensities are discussed in the next section.

It may be seen from the table that the corrections applied to the turbulent intensities were all less than 10 percent. Corrections to the rural vertical turbulent intensities were the largest. This is attributable to the larger proportion of energy at short wavelengths in the vertical spectrum. The smaller corrections to urban vertical turbulent intensities are explained by the shift of energy toward longer wavelengths in the city described in section 4.2.1.

5.1.3 Correction of Urban Lateral Turbulent Intensities

During four trials, the urban azimuth time series contained frequent discontinuities when the Vector-Vane azimuth potentiometer jumped from one end of its range to the other. Consequently, very little of the record was available for analysis. Removal of the discontinuities from the digitized data was attempted. It was not successful because noise spikes, which were not removable by automatic means, frequently accompanied the discontinuities. Because of the volume of data to be processed, hand processing was not practical. The high-frequency digitized data were used for the calculation of variances in these cases because selection of the useable data was greatly simplified. The power spectrum and turbulent intensity were calculated from the low-frequency data for the September 20 trial because few discontinuities were present.

To achieve a reasonable level of significance in the calculated variances the long-wavelength energy was filtered out by calculating the variances over sections 41 sec long. In this way energy below $1/41$ Hz (0.024 Hz) was excluded.

Numerical integration of the spectrum for the September 20 trial indicated that 30 percent of the energy was contained in wavelengths between 0.024 Hz and 0.00153 Hz. A noise peak was evident in the high-frequency power spectra computed for September 4 to 13. The noise peak was responsible for about 7 percent of the variance. The high-frequency turbulent intensities for September 4 to 13 were multiplied by 0.93 to compensate for the effect of the noise peak, and by $10/7$ to compensate for the exclusion of low-frequency energy. The corresponding correction to the turbulent intensity was an increase of 15 percent. The major uncertainty in this correction is the amount of energy below 0.024 Hz. If this amount ranged from 20 percent to 40 percent, the correction to the turbulent intensity would range from 8 percent to 25 percent. Consequently, errors of more than 10 percent in the corrected values are unlikely.

5.2 Comparison of Results

The corrected values of turbulent intensity which were obtained by the techniques previously discussed are shown in Table 10. The urban turbulent intensity was greater than the rural turbulent intensity whenever concurrent

TABLE 10

Corrected Turbulent Intensities

Date	Horizontal σ_a		Vertical σ_e		Longitudinal $\sigma_s/\bar{s}$	
	Rural	Urban	Rural	Urban	Rural	Urban
Sept. 4	0.21	0.61	0.11	0.43	0.20	0.59
5	0.22	0.43	0.09	0.37	0.20	0.70
9	..	0.37	..	0.44	..	0.91
13	0.11	0.66	0.05	0.25	0.19	0.86
17	0.10	..	0.04	..	0.14	..
20	0.14	0.51	0.07	0.34	0.21	0.45
average	0.16	0.52	0.08	0.37	0.19	0.70

observations were available. The ratio of the average urban turbulent intensity to the average rural turbulent intensity was 4.6 for the vertical component, 3.7 for the longitudinal component, and 3.2 for the lateral component.

CHAPTER VI

CONCLUSIONS

Turbulent intensities and power spectra at an urban and a rural site were measured in conditions that were generally stable with light winds. The scales and intensities of turbulence were compared and some systematic differences were found. Some aspects of the power spectra were compared to the predictions of theory. The conclusions which were drawn are summarized below.

The intensity of turbulence was found to be more than three times as great at the urban site as at the rural site.

A $-5/3$ power law fitted the urban power spectra of wind speed, elevation angle, and azimuth angle well at short wavelengths. The power law fitted these spectra at wavelengths shorter than 8 to 50 times the distance to the ground.

Rural speed spectra were also well represented by a $-5/3$ law. The long-wavelength limit was 10 to 30 times the height above ground. Rural elevation spectra did not agree with a $-5/3$ law over as wide a range of wavelengths as did speed spectra or urban elevation spectra. The rural elevation spectra also exhibited more scatter in the region where agreement was obtained. The long-wavelength limit for

rural elevation spectra was 3 to 8 times the height. Rural azimuth spectra agreed poorly with a $-5/3$ law. However, the rural azimuth signals were the smallest recorded, and it is likely that the signal-to-noise ratio was higher.

A $-5/3$ law was observed to agree well with the shape of many spectra at wavelengths long in comparison to the height above the ground. Also, a $-5/3$ law applied to rural speed spectra over a range of 1 to 4 octaves when it did not apply to rural elevation spectra. It was concluded that isotropy is not a necessary condition in order that a $-5/3$ law should apply to atmospheric turbulence spectra.

The peak of the elevation angle spectrum occurred at longer wavelengths at the urban site than at the rural site. The peak of the rural elevation spectrum agreed with the near-neutral value of the universal function proposed for vertical velocity spectra by Busch and Panofsky (1968). This function was obtained on the basis of similarity theory and several sets of recent measurements at different locations. The peak of the urban elevation angle spectrum was an octave and a half lower than predicted by Busch and Panofsky. Two alternative hypotheses were proposed to explain the discrepancy.

The first alternative is that vertical velocity spectra do not obey similarity theory close to the ground in a rough urban environment. That is, the vertical scale of turbulence in the city decreases more slowly with height than the linear relationship required by similarity theory.

It should be noted that observations in the city were made at a height small in comparison to the height of the major roughness elements. Consequently, an important assumption underlying the derivation of the similarity theory does not apply.

The second alternative is that the vertical velocity spectrum and the elevation angle spectrum are not similar in shape when the turbulent intensity is high. However, the agreement of the polar turbulence spectra with a $-5/3$ law suggests that the same relationships hold for polar and Cartesian spectra, at least at short wavelengths.

Investigation of this problem is outside the scope of the present study. The literature provides little information on the relationship between turbulence statistics in polar coordinates and their Cartesian counterparts. Research on this topic would be of considerable interest. Until this is done, the first alternative hypothesis must be only tentative.

REFERENCES

REFERENCES

- Bendat, J. S., and A. G. Piersol, 1966: Measurement and Analysis of Random Data. New York, Wiley & Sons, 390 pp.
- Bowne, N. E., J. T. Ball, and G. E. Anderson, 1968: Some Measurements of the Atmospheric Boundary Layer in an Urban Complex. Final Report, Vol. I, Contract No. DAAD09-67-C-0108(R), The Travelers Research Center, Inc., 95 pp.
- Bowne, N. E., and J. T. Ball, 1970: Observational comparison of rural and urban boundary layer turbulence. J. Appl. Meteor., 9, 862 - 873.
- Busch, N. E., and H. A. Panofsky, 1968: Recent spectra of atmospheric turbulence. Quart. J. Roy. Meteor. Soc., 94, 132 - 148.
- Canada, Department of the Environment, Atmospheric Environment Service. Meteorological Tower Bulletin. Fall, 1970.
- Cochran, W. T., J. W. Cooley, D. L. Favon, H. D. Helms, R. A. Kaenel, W. W. Lang, G. C. Maling, Jr., D. E. Nelson, C. M. Rader, and P. D. Welch, 1967: What is the fast Fourier transform? IEEE Transactions on Audio and Electroacoustics, AU - 15, no. 2, 45 - 55.
- Davenport, A. G., 1967: Instrumentation and measurements of wind speed spectra in a city. Proceedings of the First Canadian Conference on Micrometeorology. Toronto; Canada, Department of Transport, Meteorological Branch, Part II, 361 - 368.
- Hage, K. D., and R. W. Longley, 1970: Ventilation and mixing in Alberta cities. Man and His Environment, Proceedings of the First Banff Conference on Pollution. Edited by M. A. Ward, Oxford, Pergamon Press, 99 - 113.
- Hay, J. S., and F. Pasquill, 1957: Diffusion from a fixed source at a height of a few hundred feet in the atmosphere. J. Fluid Mechanics, 2, 299 - 310.

- Kolmogorov, A. N., 1941: The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers. Akademiia Nauk SSSR, Doklady, 30, 301 - 305.
- Lumley, J. L., and H. A. Panofsky, 1964: The Structure of Atmospheric Turbulence. New York, Interscience Publishers, 239 pp.
- Macready, P. B., Jr., and H. R. Jex, 1964: Response characteristics and meteorological utilization of propellor and vane wind sensors. J. Appl. Meteor., 3, 182 - 193.
- Monin, A. S., and A. M. Obukhov, 1954: Basic laws of turbulent mixing in the ground layer of the atmosphere. Akademiia Nauk SSSR, Leningrad, Geofizicheskii Institut, Trudy, 24(151), 163 - 187. (English translation obtainable from Special Libraries Association Translation Center.)
- Munn, R. E., 1966: Descriptive Micrometeorology. New York, Academic Press, 245 pp.
- Panofsky, H. A., and R. A. McCormick, 1954: Properties of spectra of atmospheric turbulence at 100 meters. Quart. J. Roy. Meteor. Soc., 80, 546 - 564.
- Van der Hoven, I., 1957: Power spectrum of horizontal wind speed in the frequency range from 0.0007 to 900 cycles per hour. J. Meteor., 14, 160 - 164.
- Walker, E. R., 1964: Atmospheric turbulence characteristic measured at Suffield Experimental Station. Beitr. Phys. Atmos., 37, 38 - 52.
- Welch, P. D., 1967: The use of the fast Fourier transform for the estimation of power spectra: a method based on time averaging over short, modified, periodograms. IEEE Transactions on Audio and Electroacoustics, AU - 15, no. 2, 70 - 73.

APPENDIX

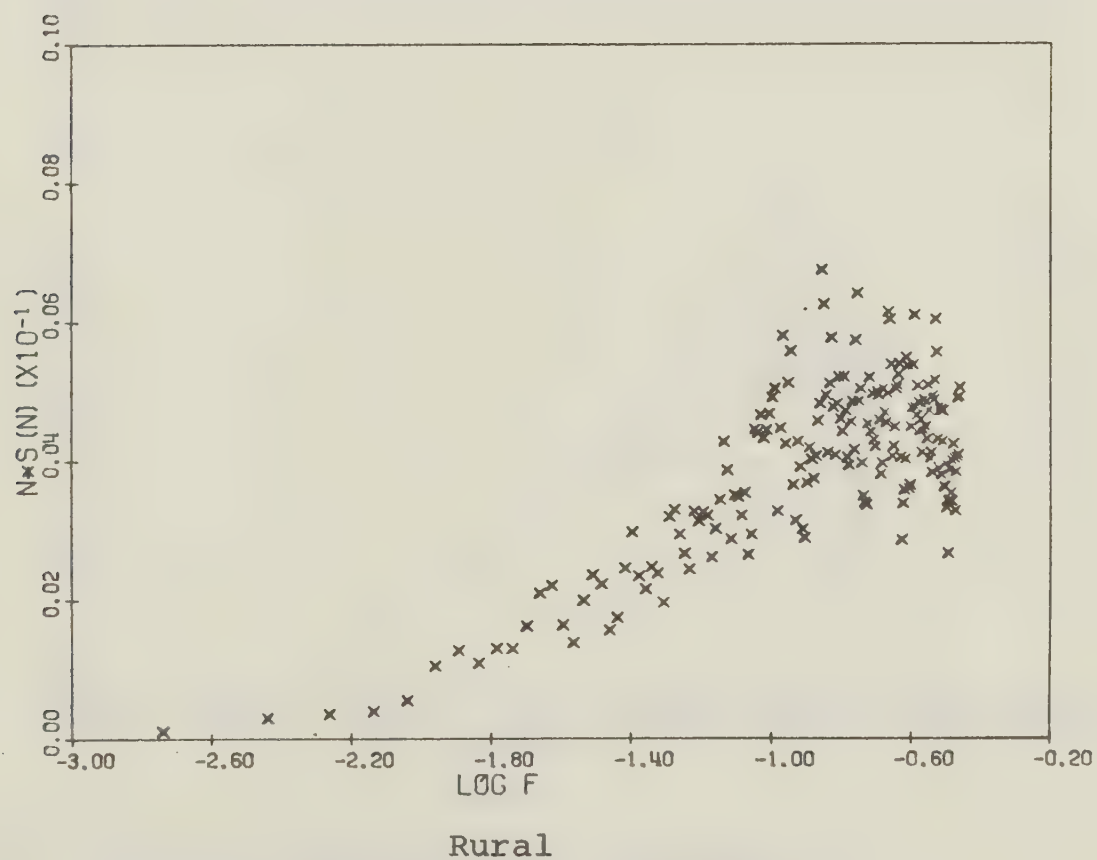
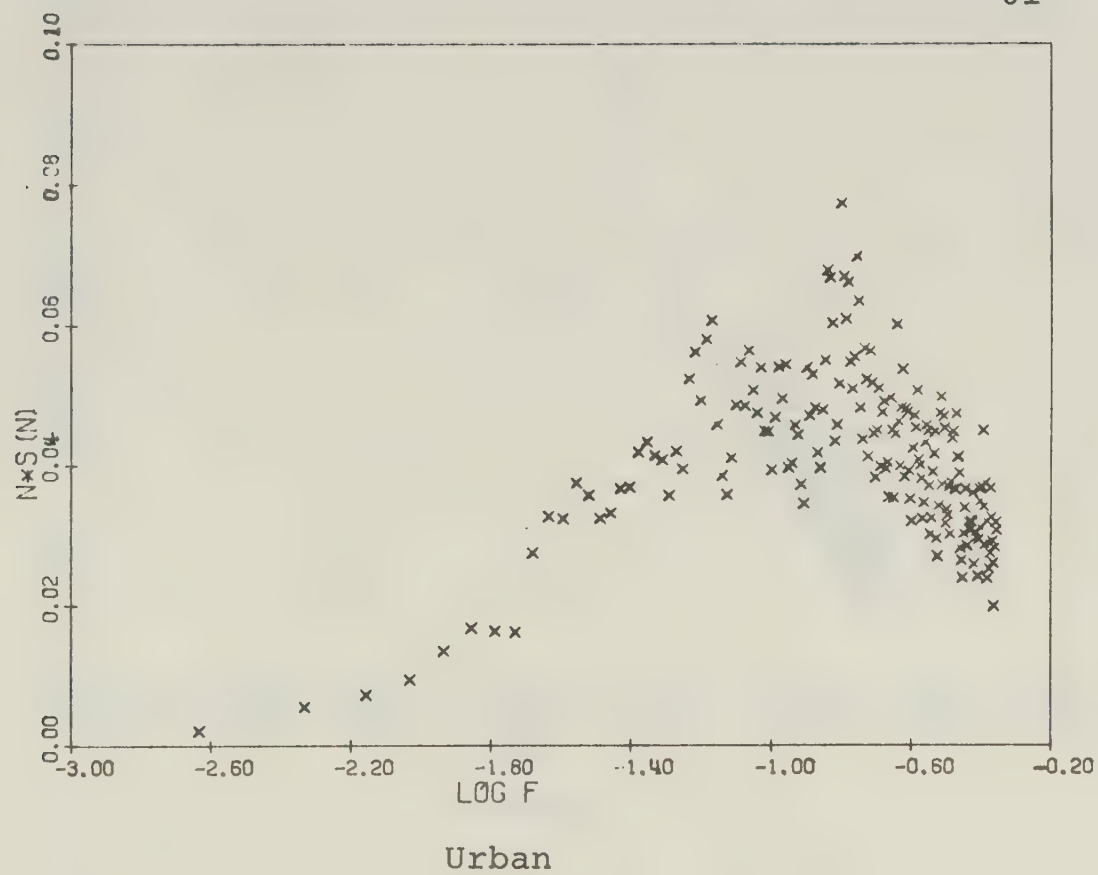


Figure A-1 Elevation angle spectra, September 4
($n \cdot S(n)$ vs $\log f$)

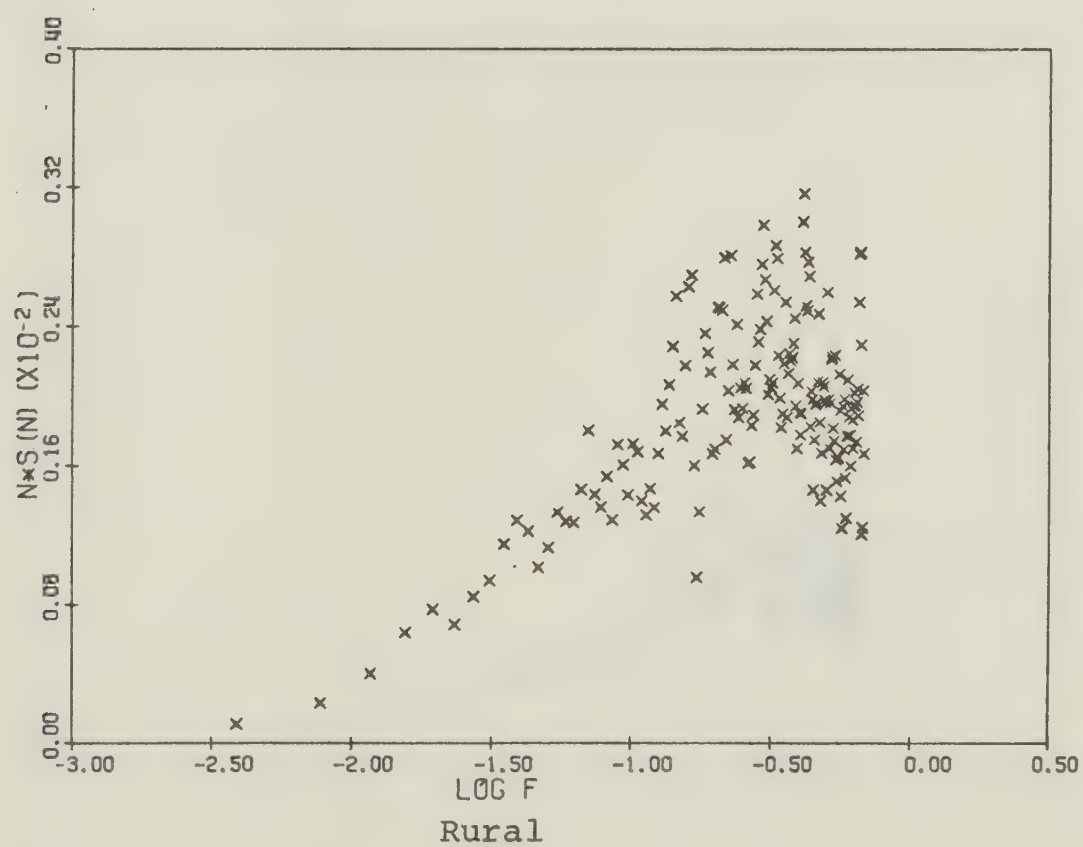
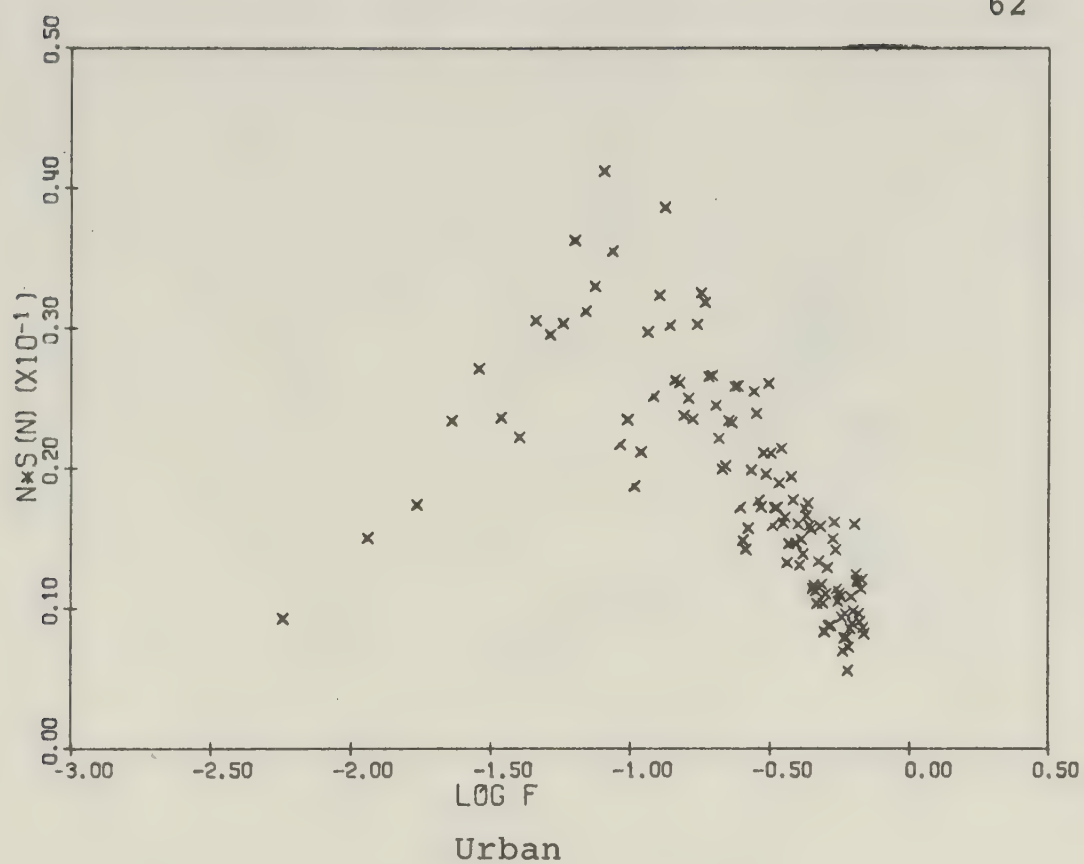


Figure A-2 Elevation angle spectra, September 5
($n \cdot S(n)$ vs $\log f$)

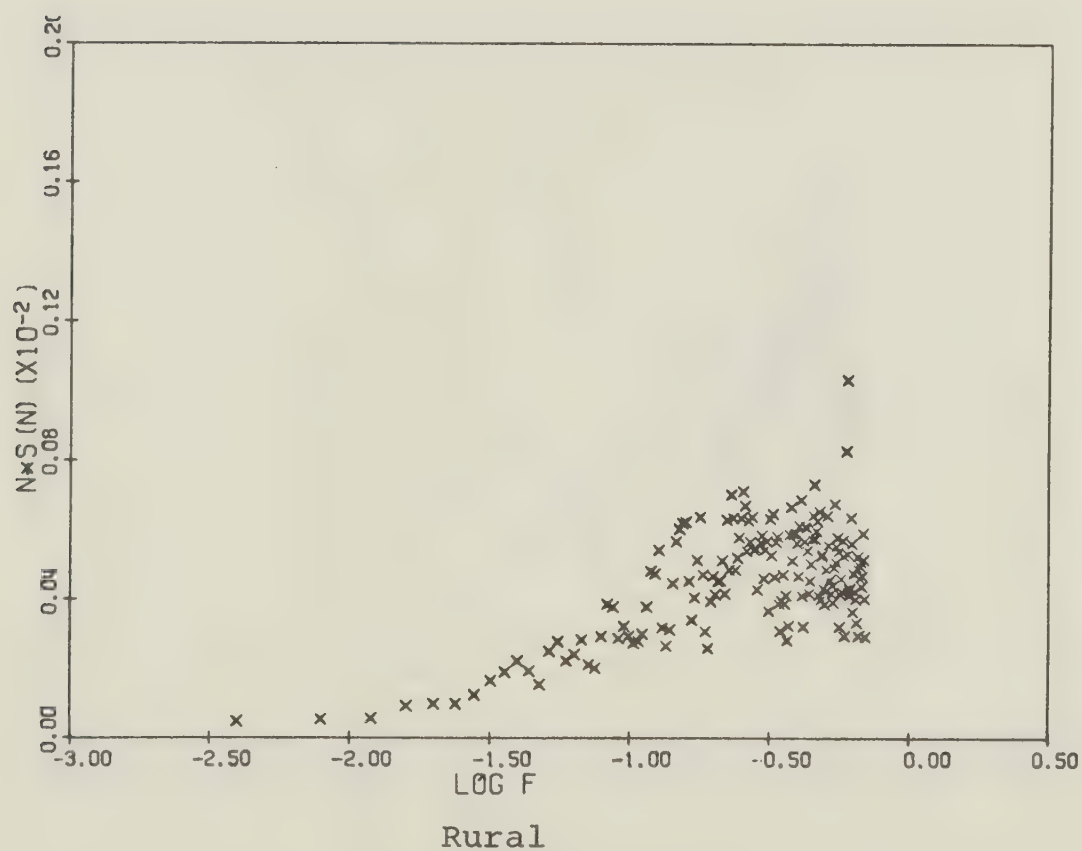
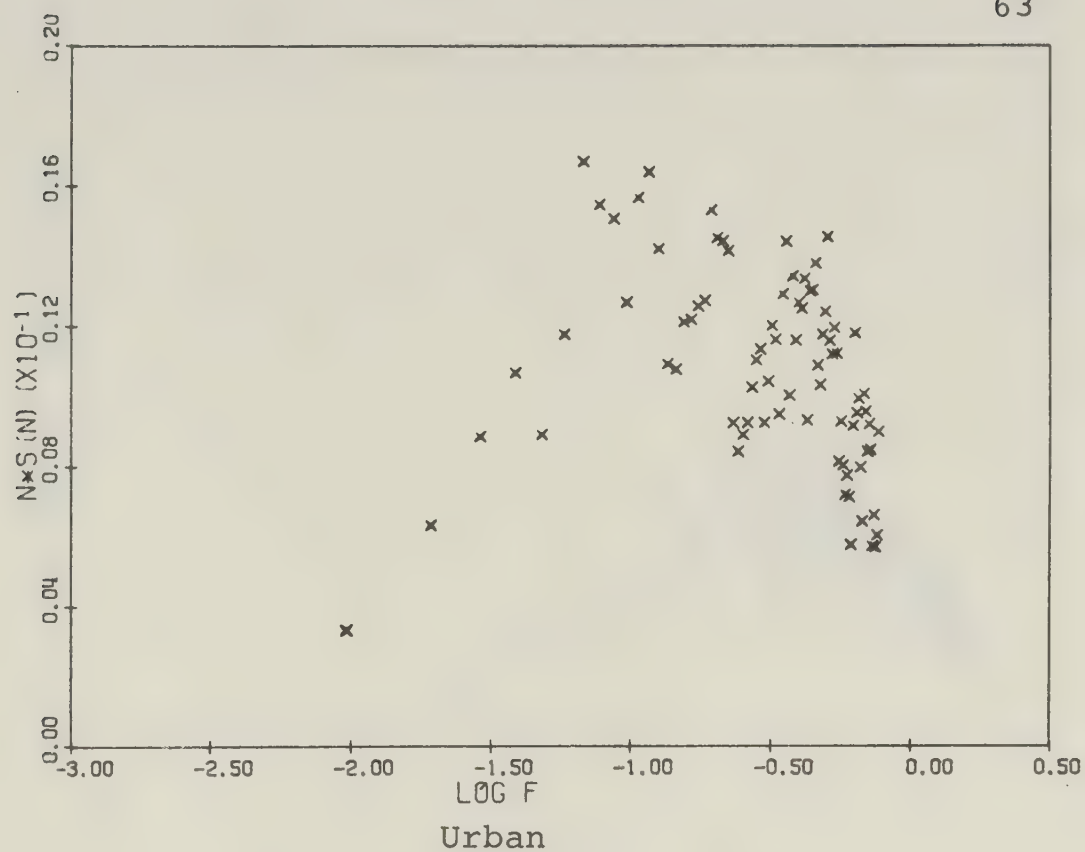


Figure A-3 Elevation angle spectra, September 13
($n \cdot S(n)$ vs $\log f$)

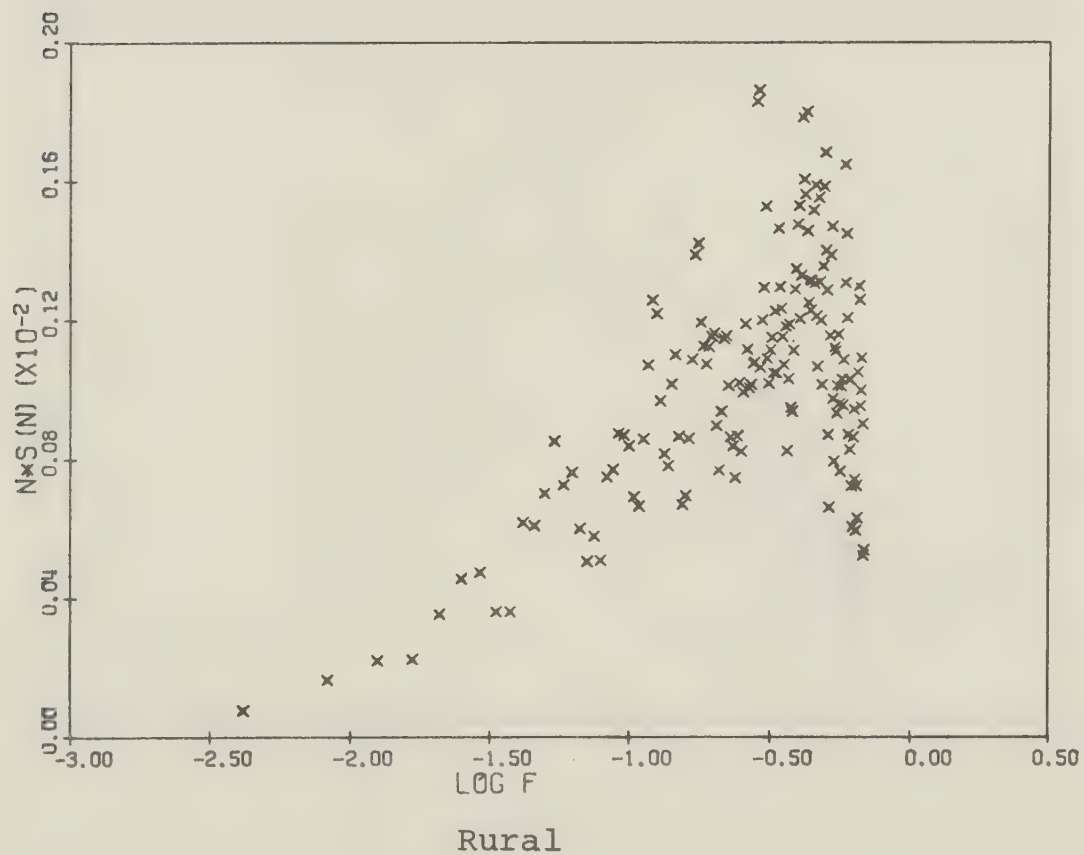
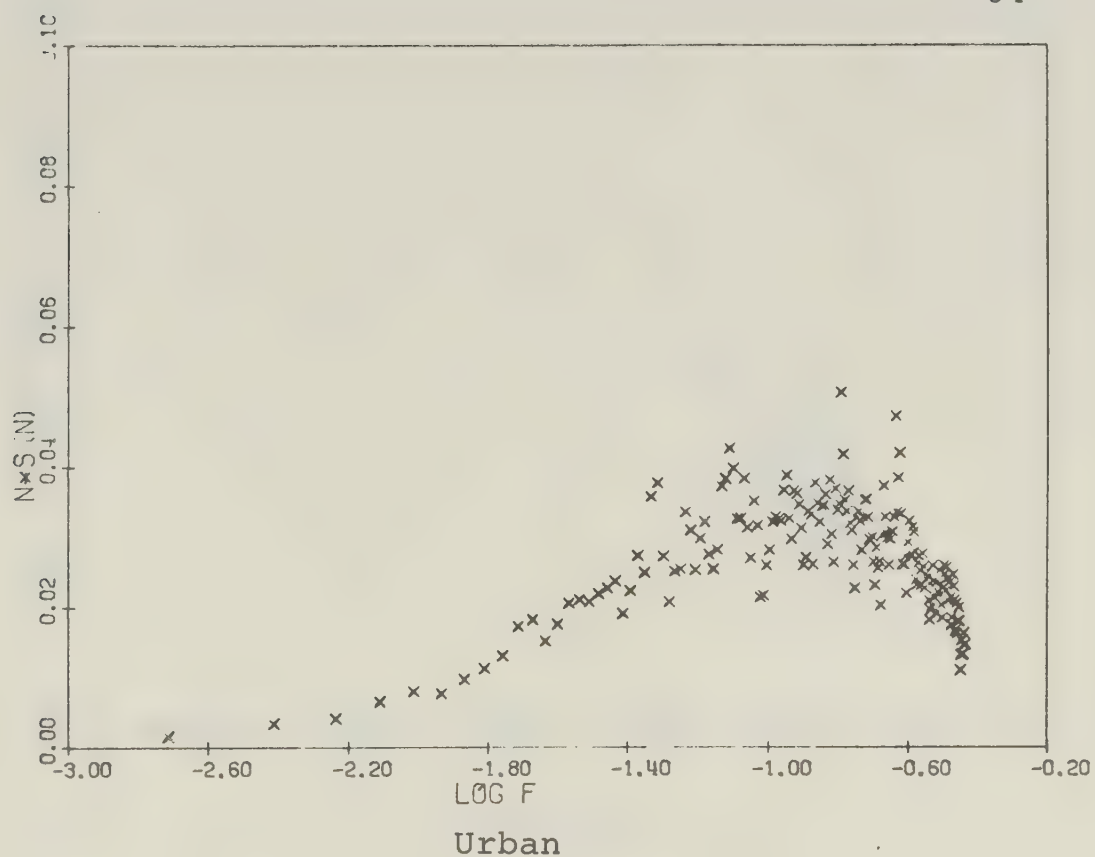


Figure A-4 Elevation angle spectra, September 20
($n \cdot S(n)$ vs $\log f$)

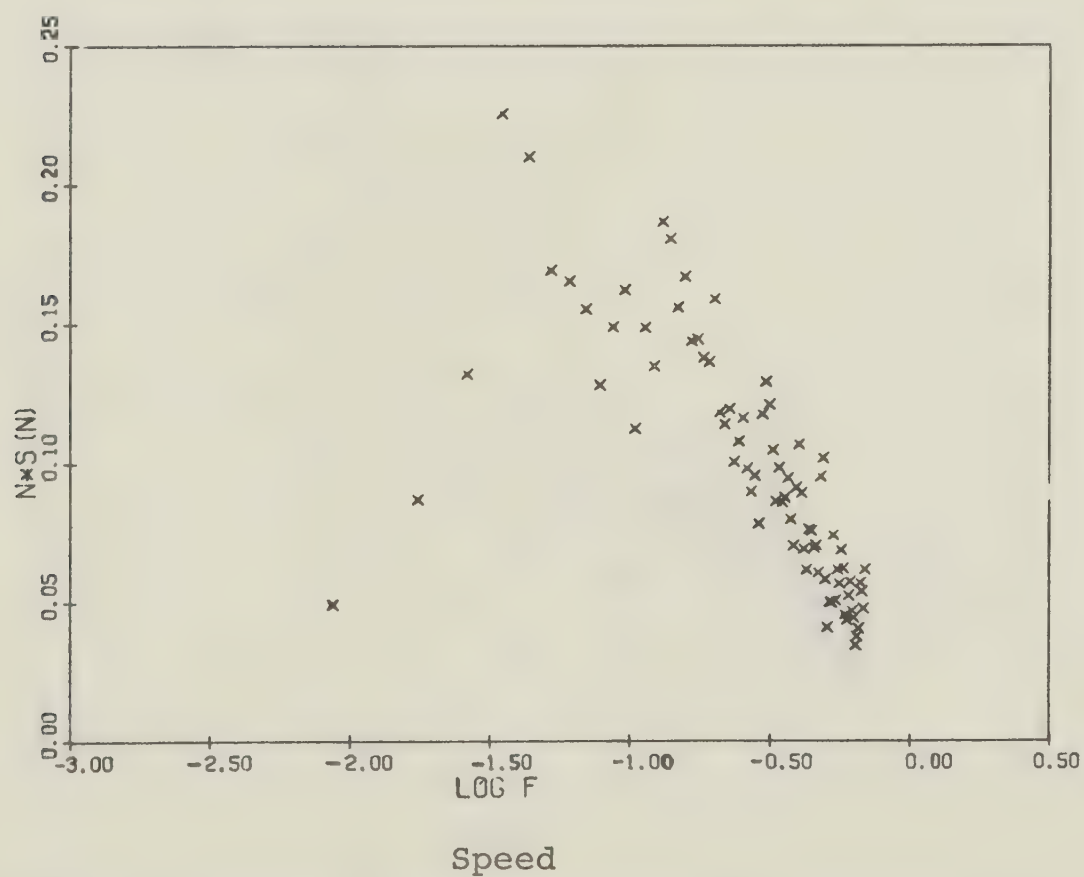
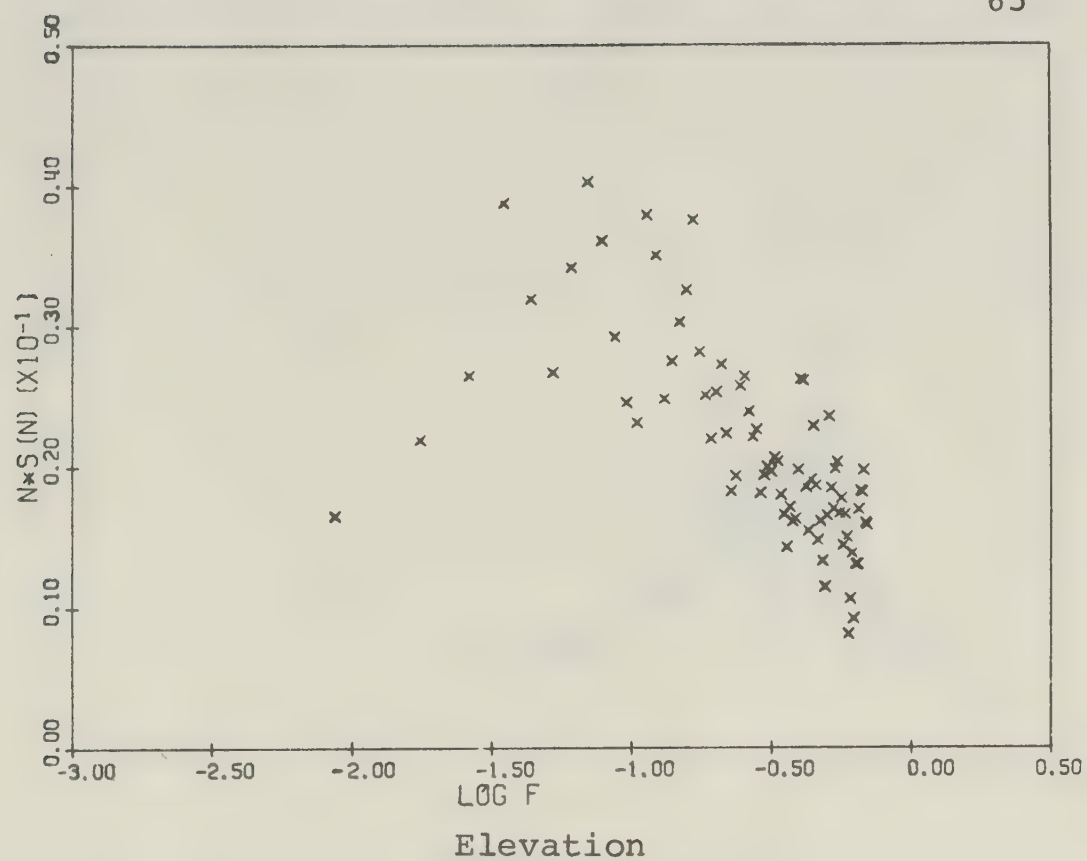


Figure A-5 Urban speed and elevation spectra, September 9
($n \cdot S(n)$ vs $\log f$)

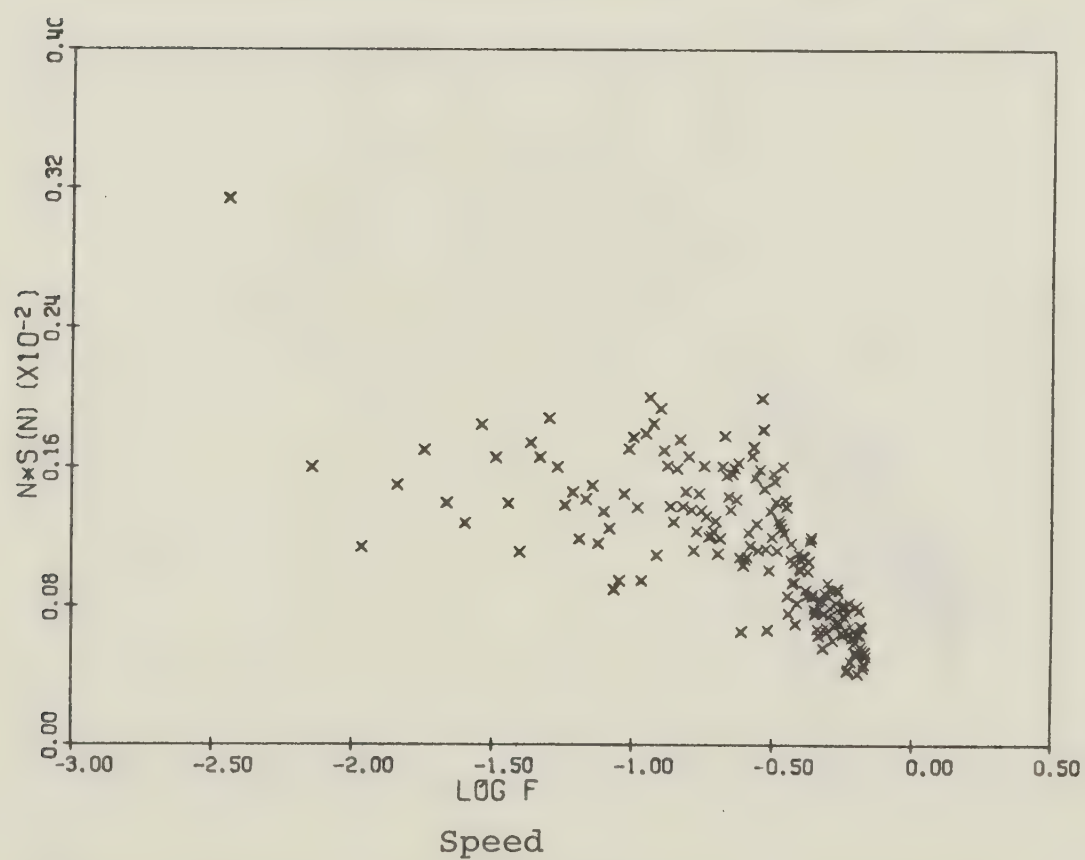
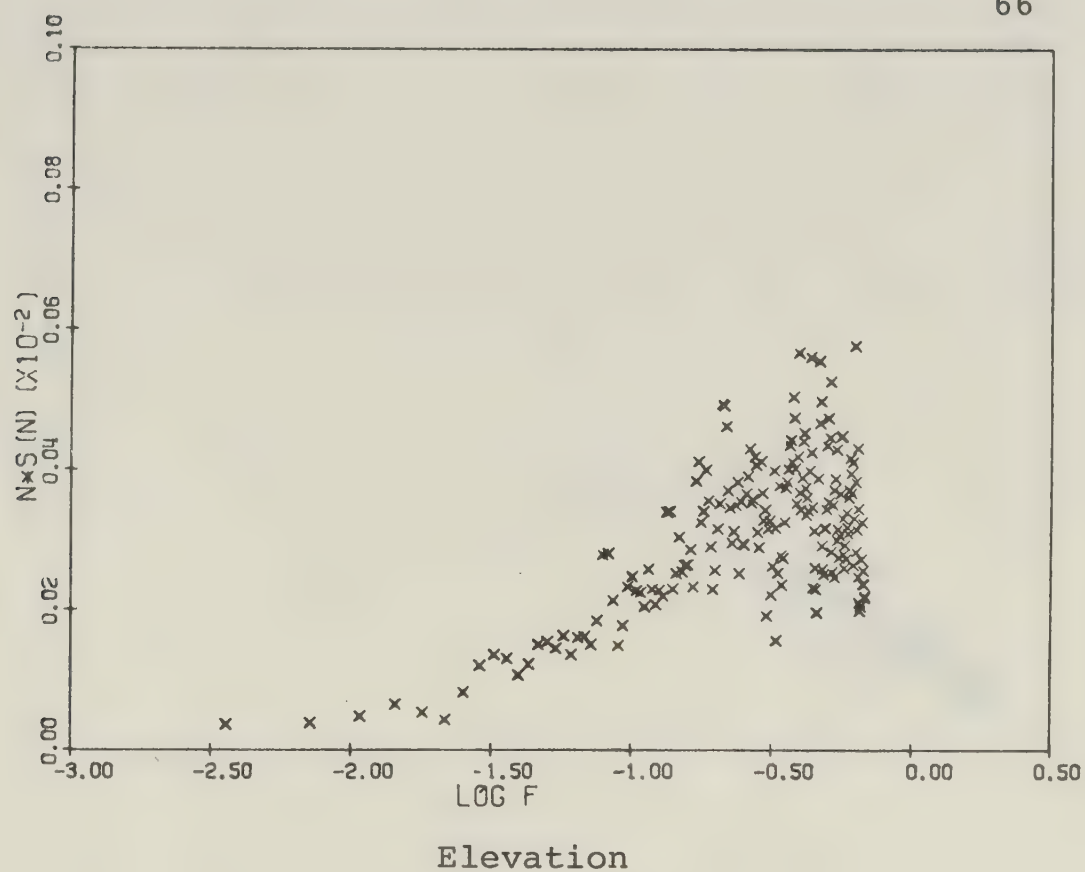


Figure A-6 Rural speed and elevation spectra, September 17
($n \cdot S(n)$ vs $\log f$)

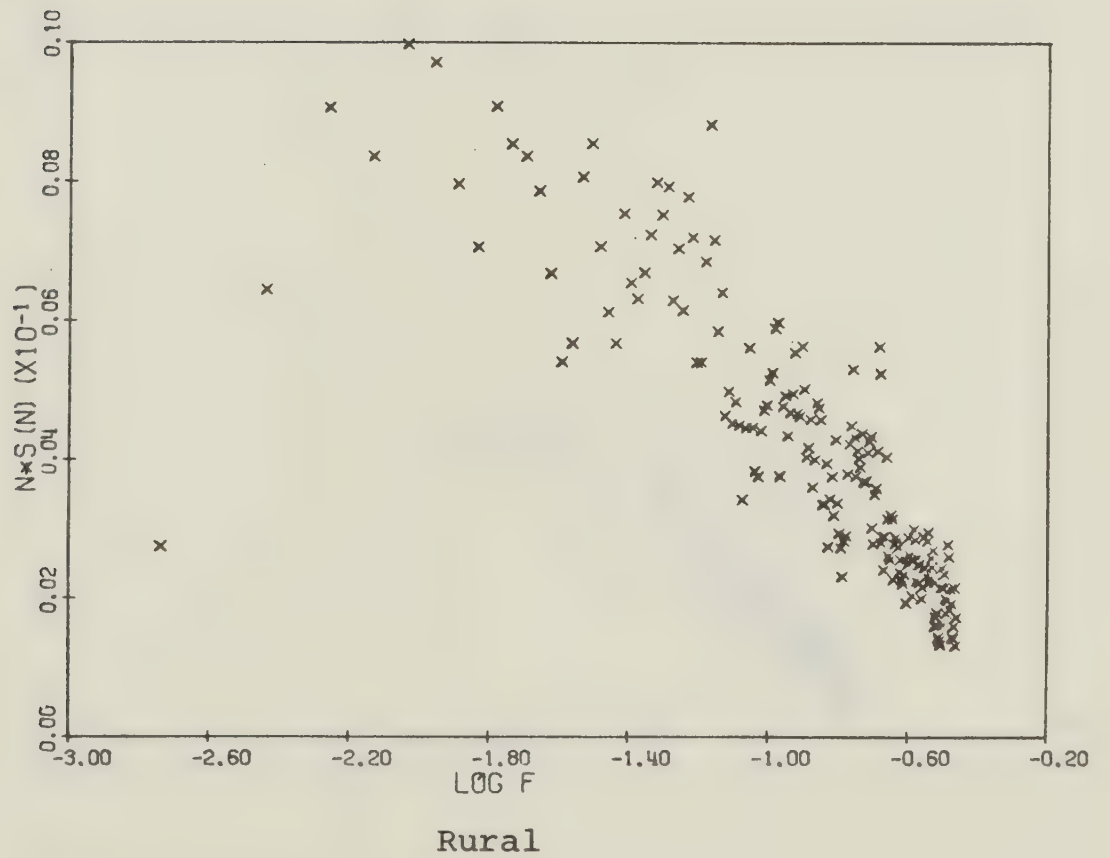
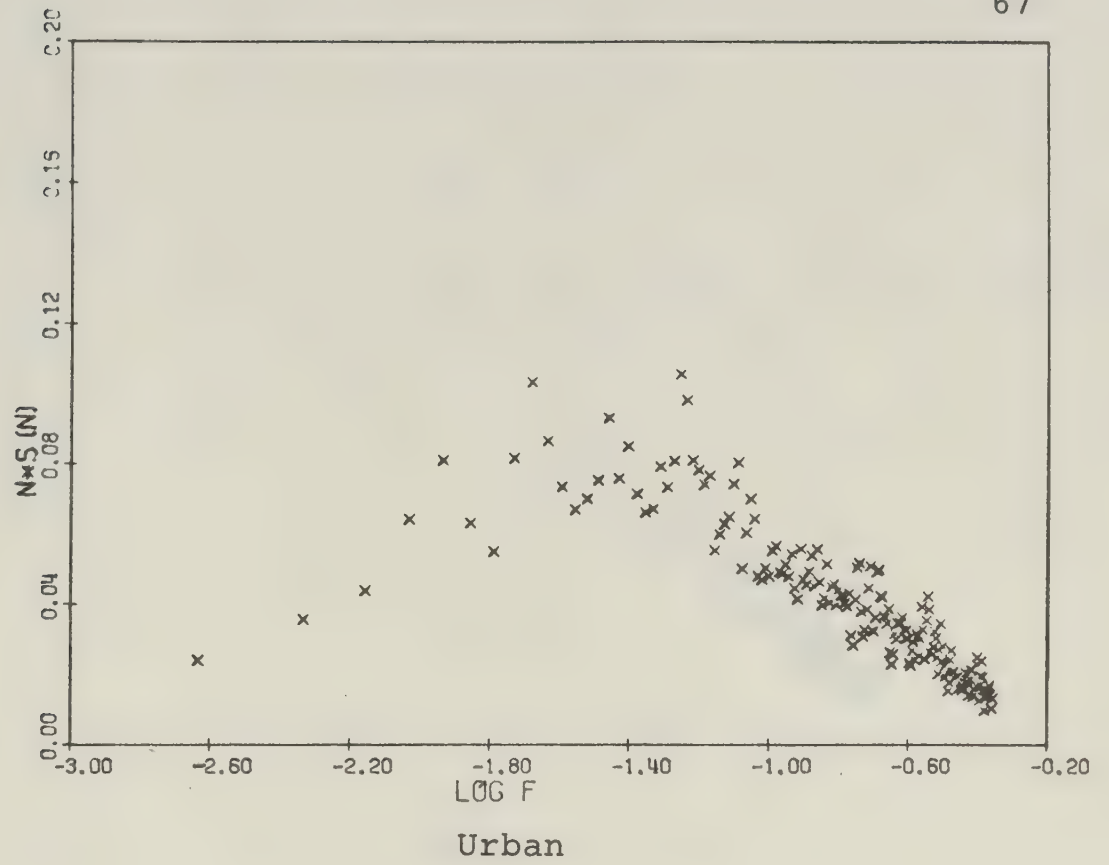


Figure A-7 Speed spectra, September 4
($n \cdot S(n)$ vs $\log f$)

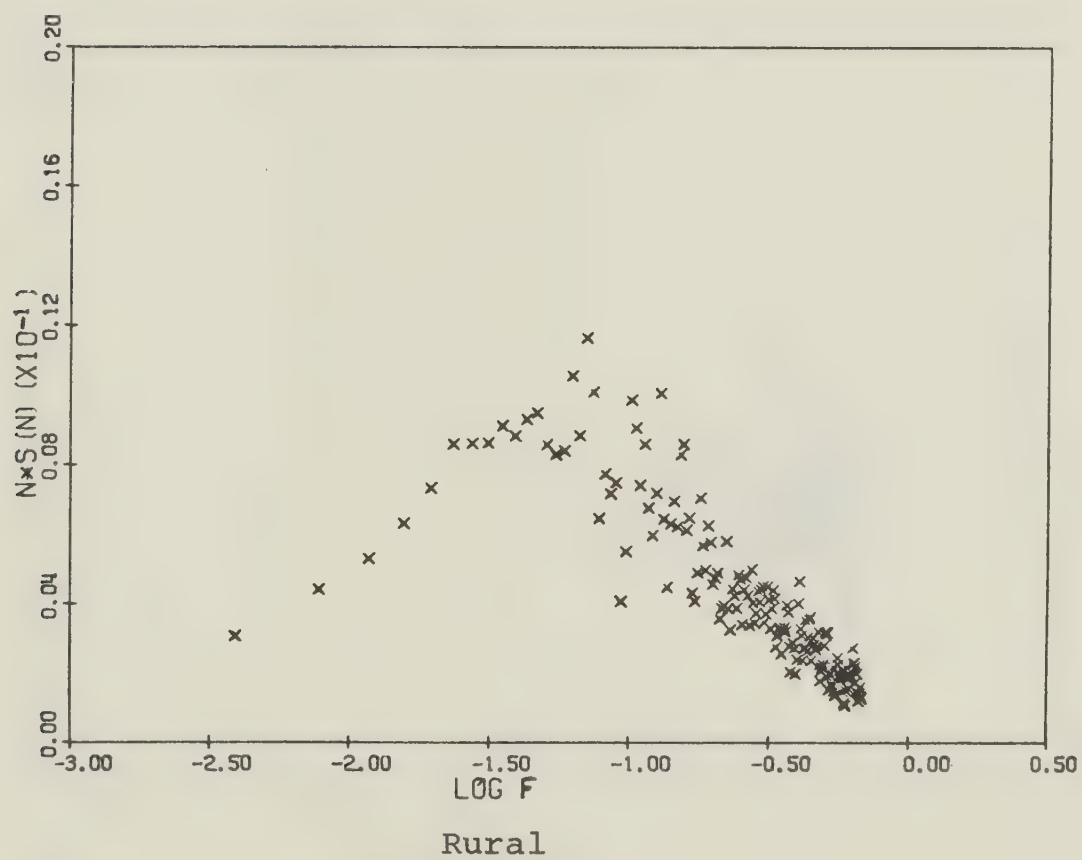
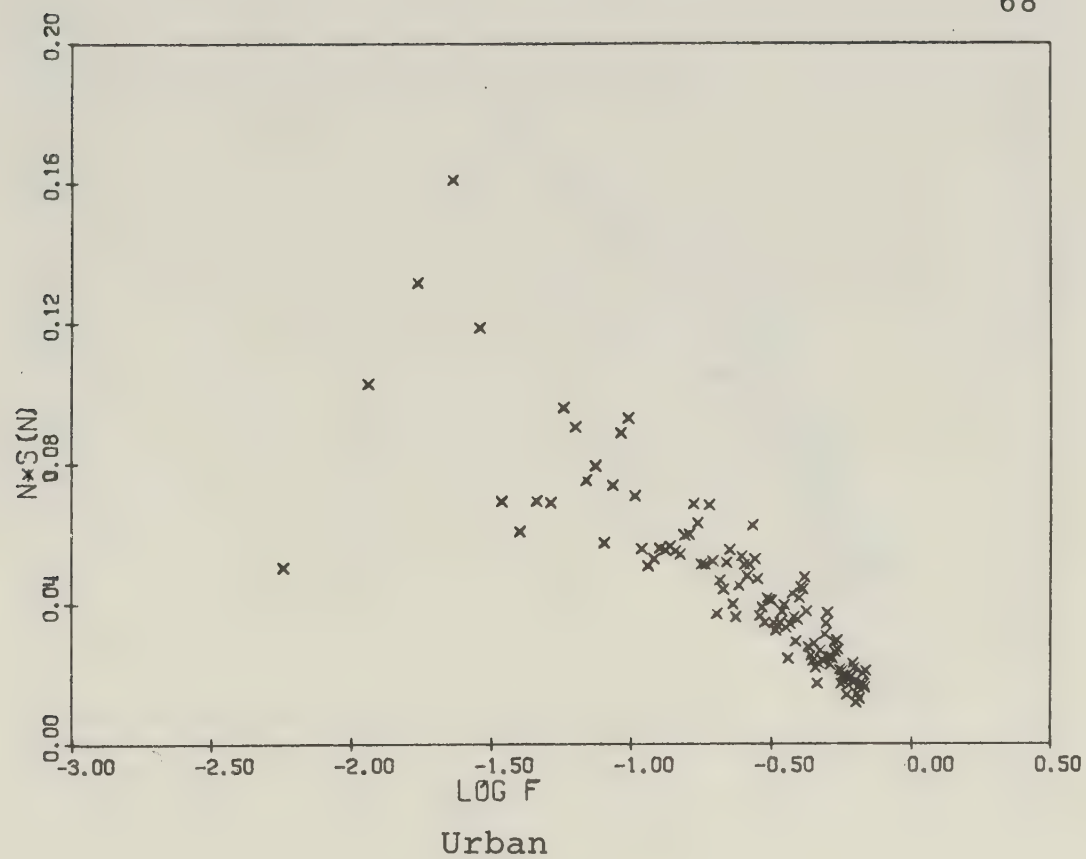


Figure A-8 Speed spectra, September 5
($n \cdot S(n)$ vs $\log f$)

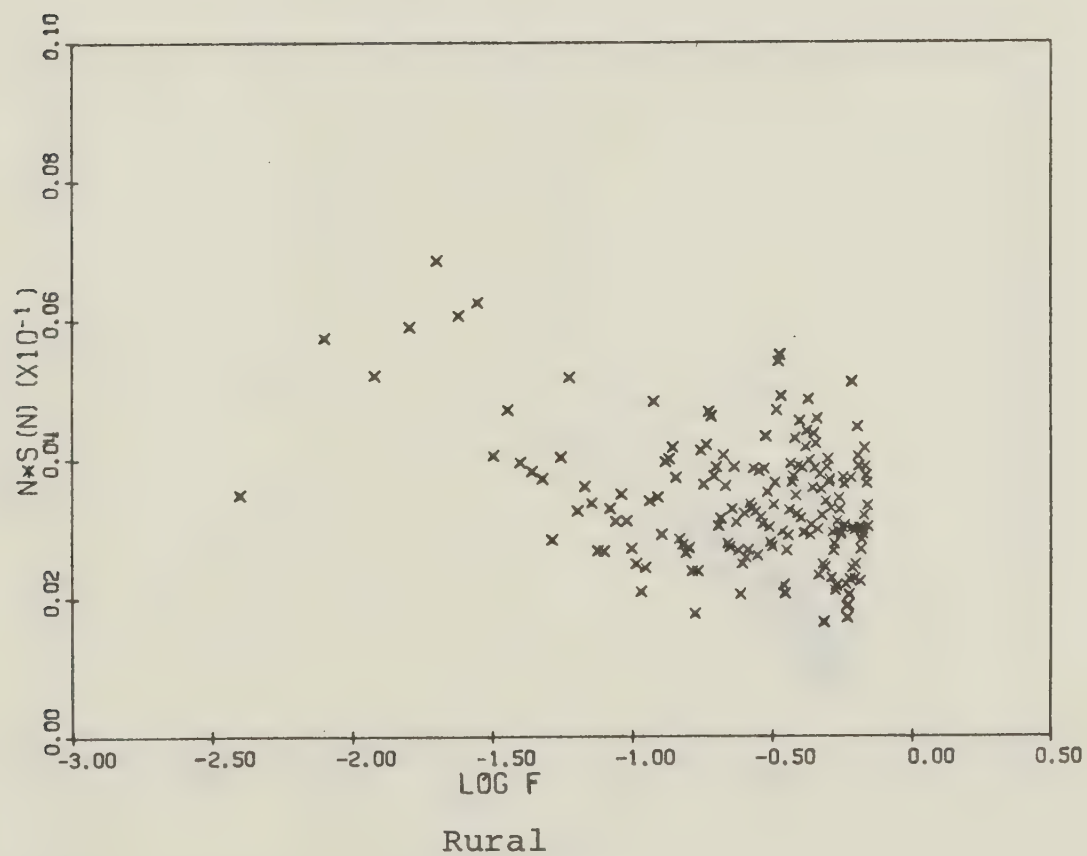
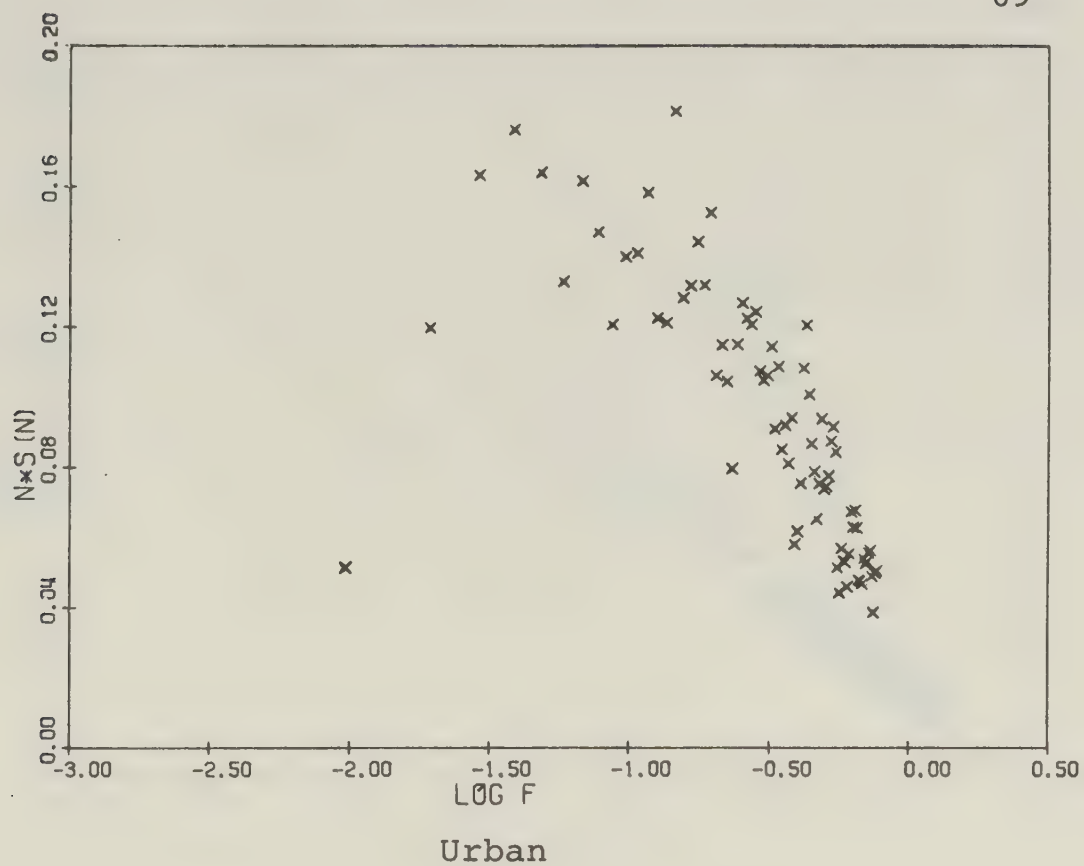


Figure A-9 Speed spectra, September 13
($n \cdot S(n)$ vs $\log f$)

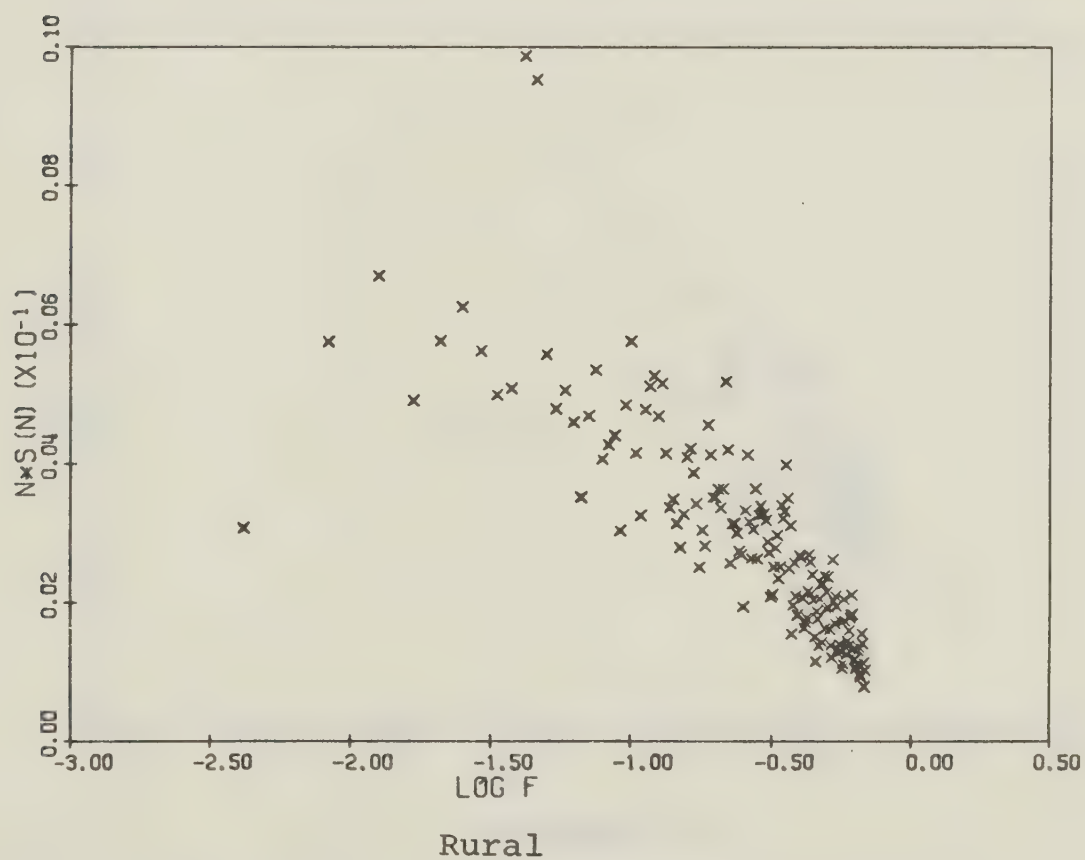
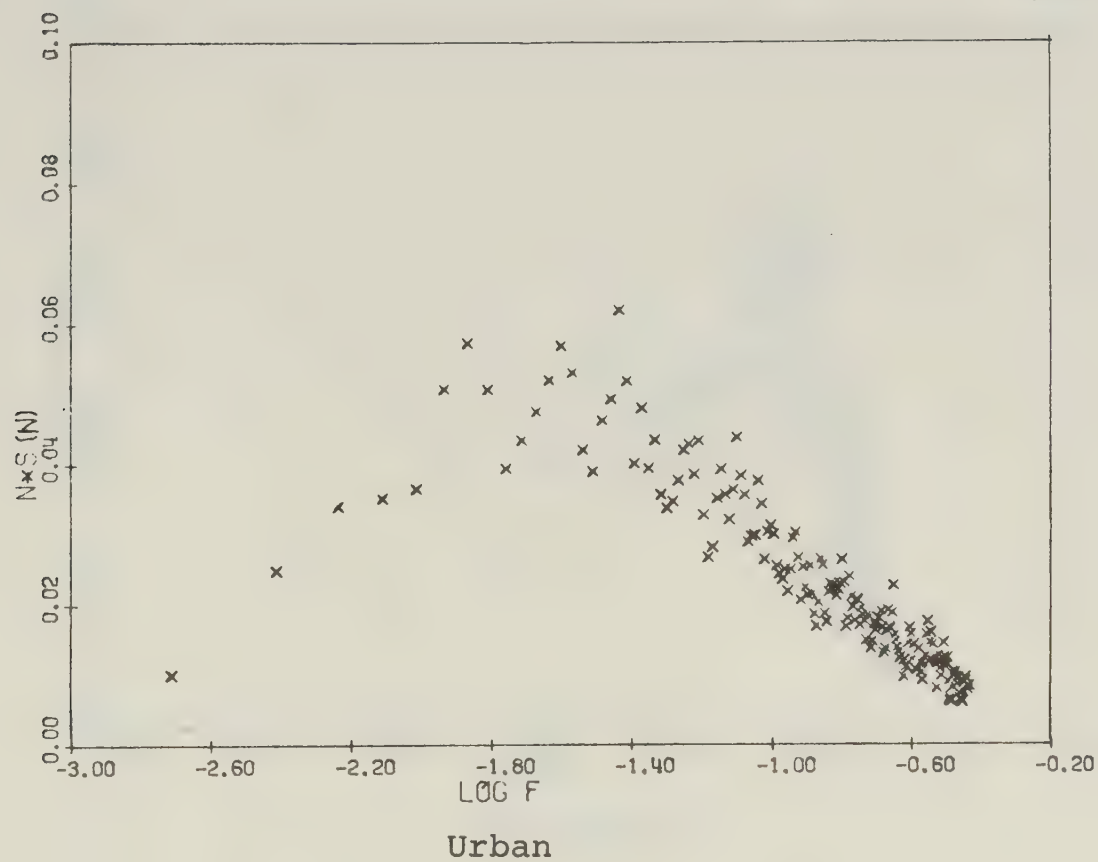
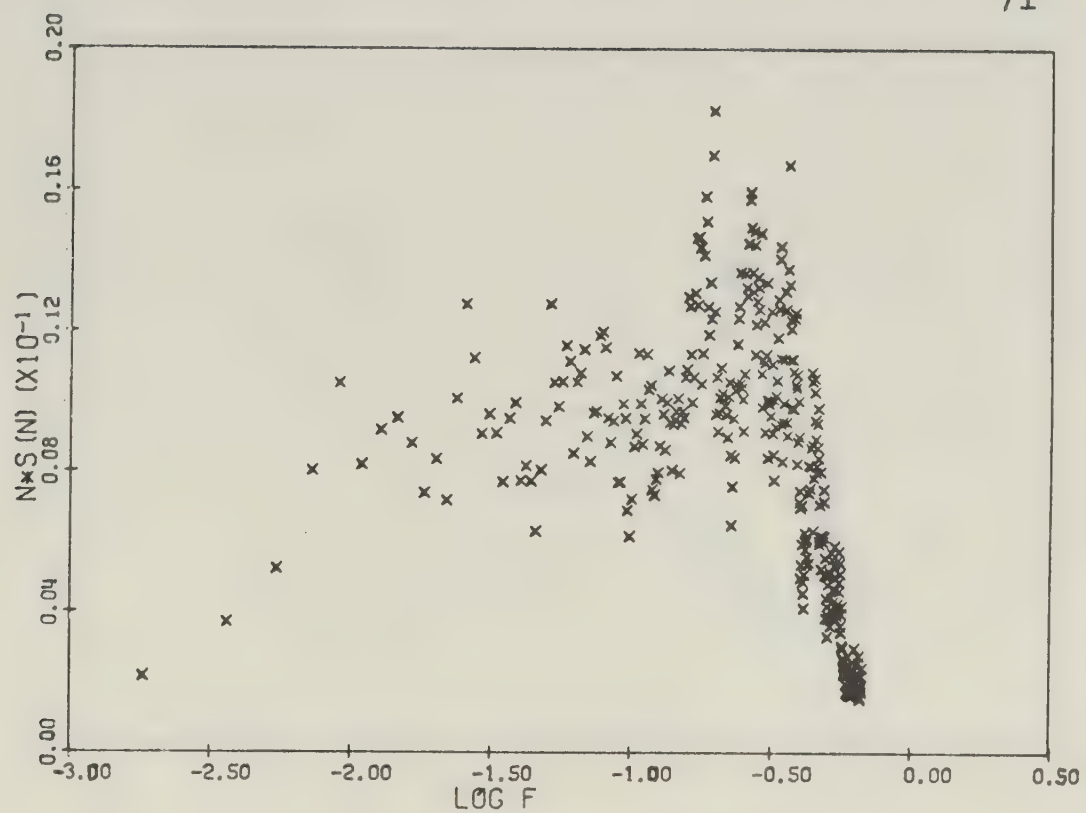
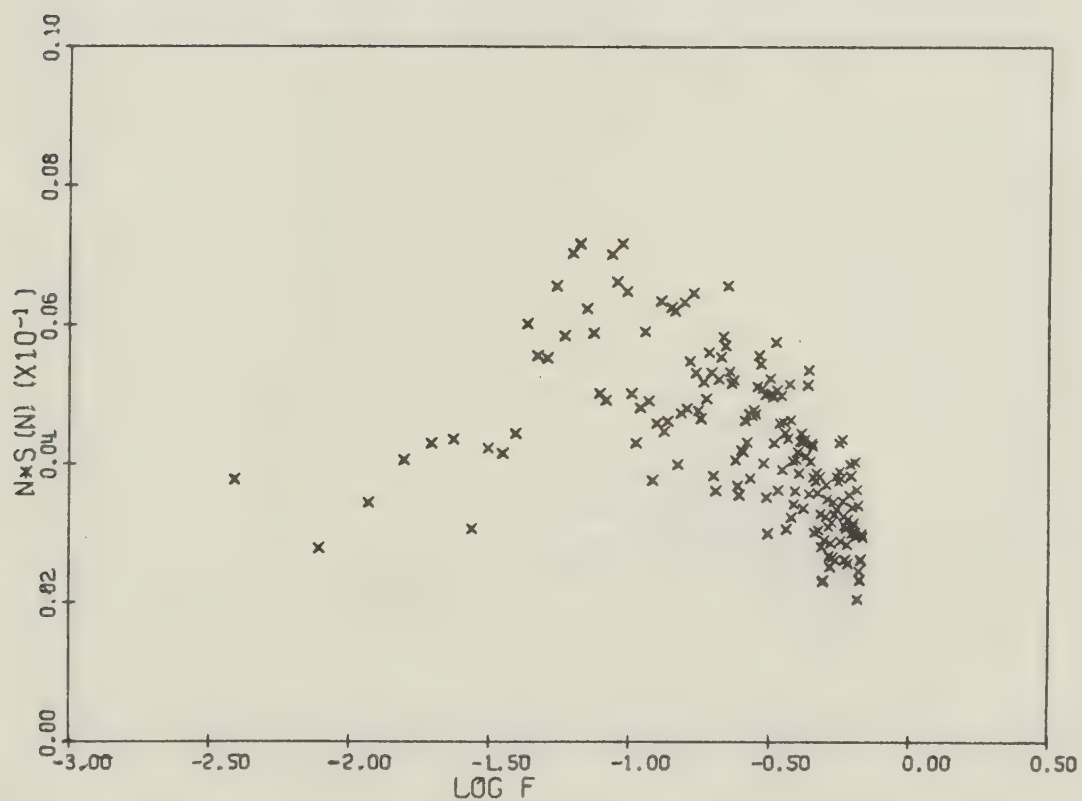


Figure A-10 Speed spectra, September 20
($n \cdot S(n)$ vs $\log f$)



September 4



September 5

Figure A-11 Rural azimuth spectra, September 4 and 5
($n \cdot S(n)$ vs $\log f$)

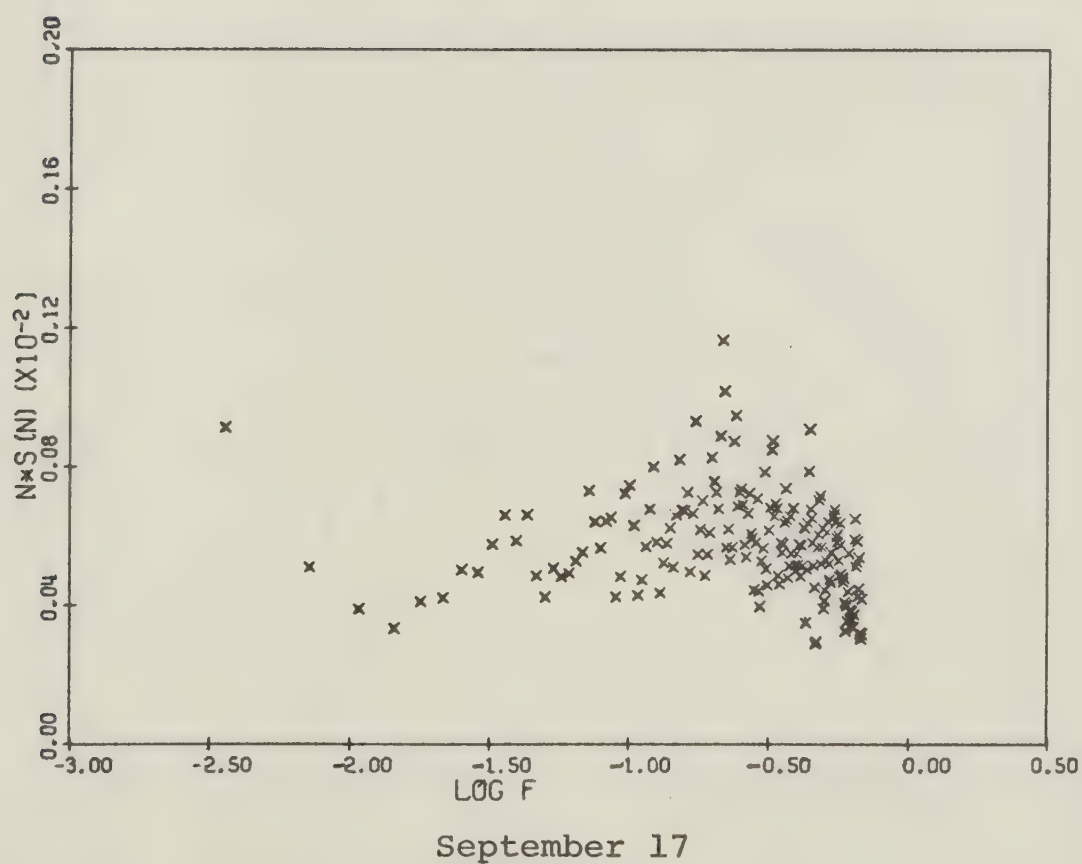
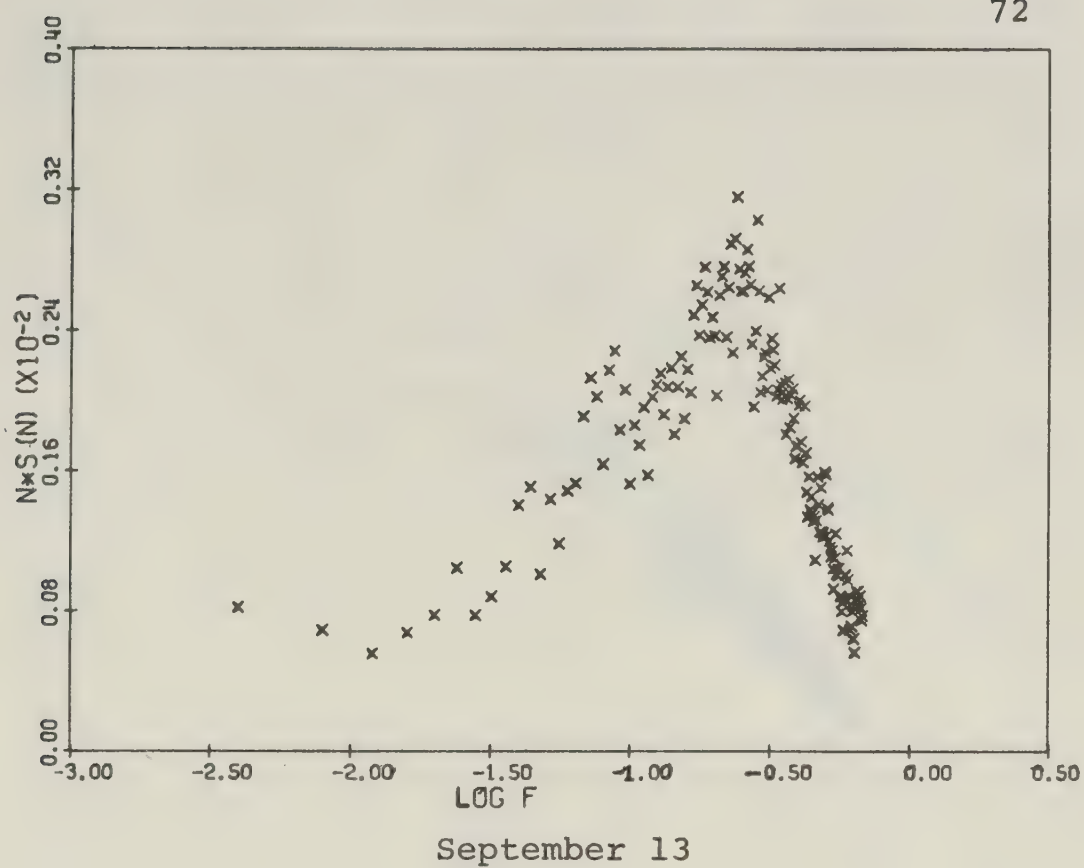
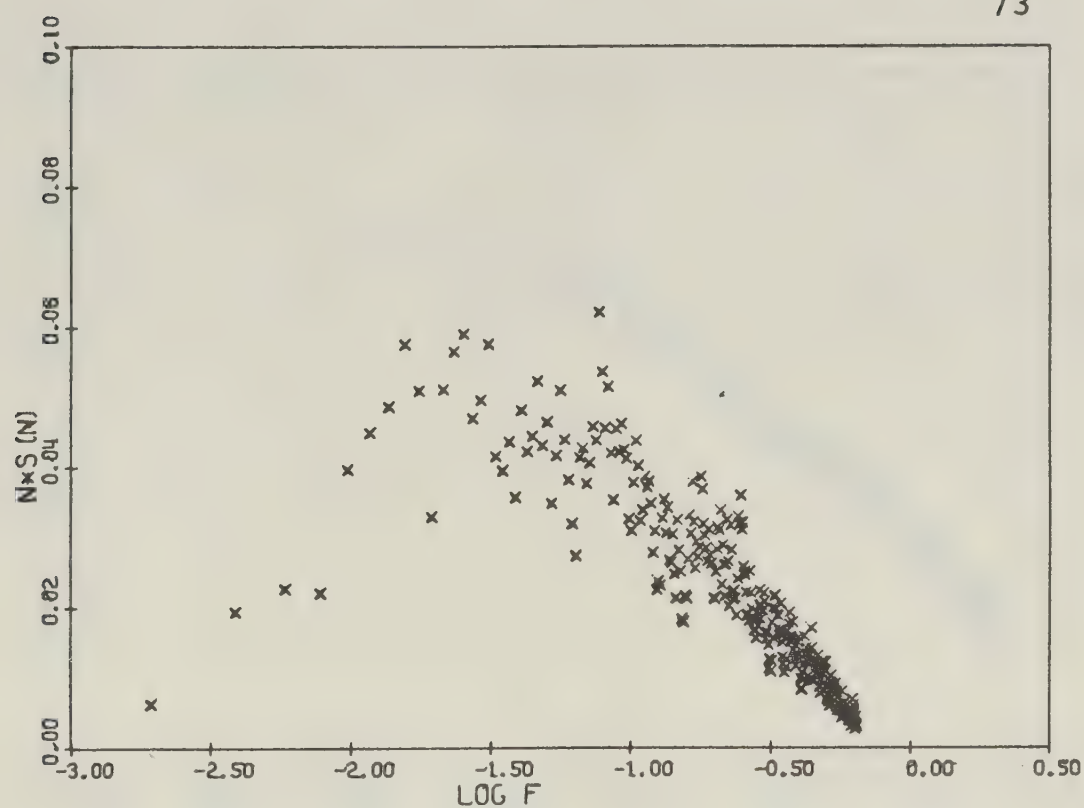
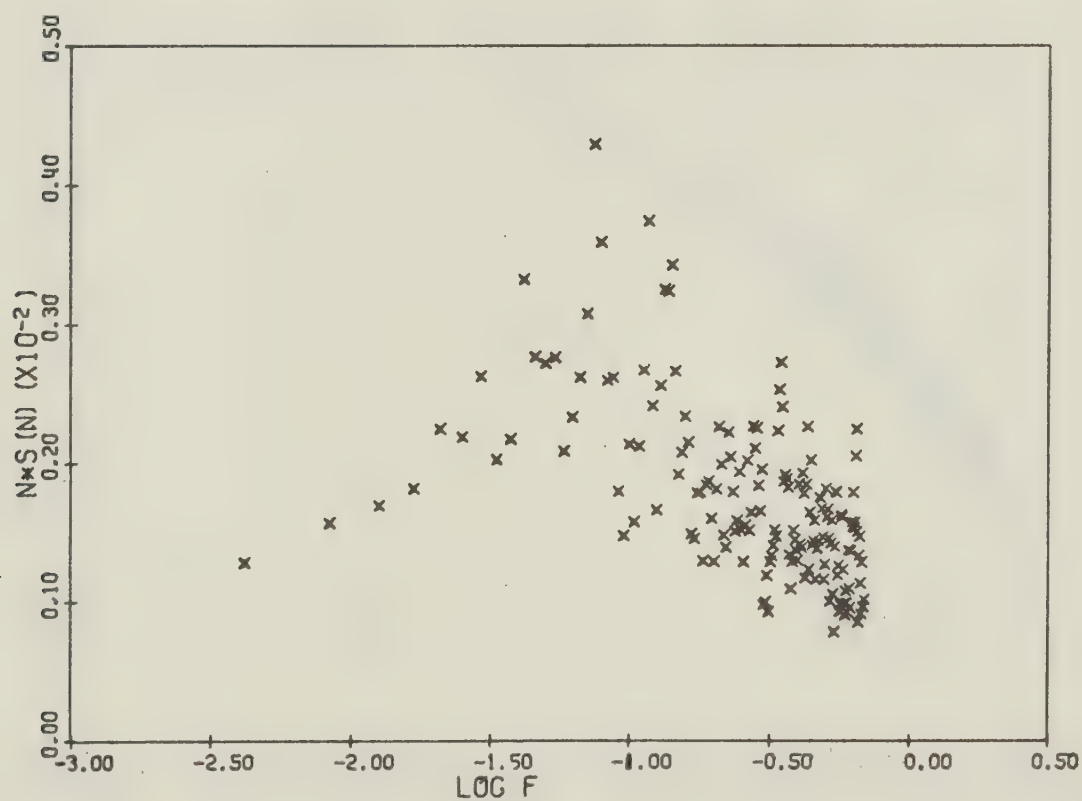


Figure A-12 Rural azimuth spectra, September 13 and 17
($n \cdot S(n)$ vs $\log f$)



Urban



Rural

Figure A-13 Urban and rural azimuth spectra, September 20
($n \cdot S(n)$ vs $\log f$)

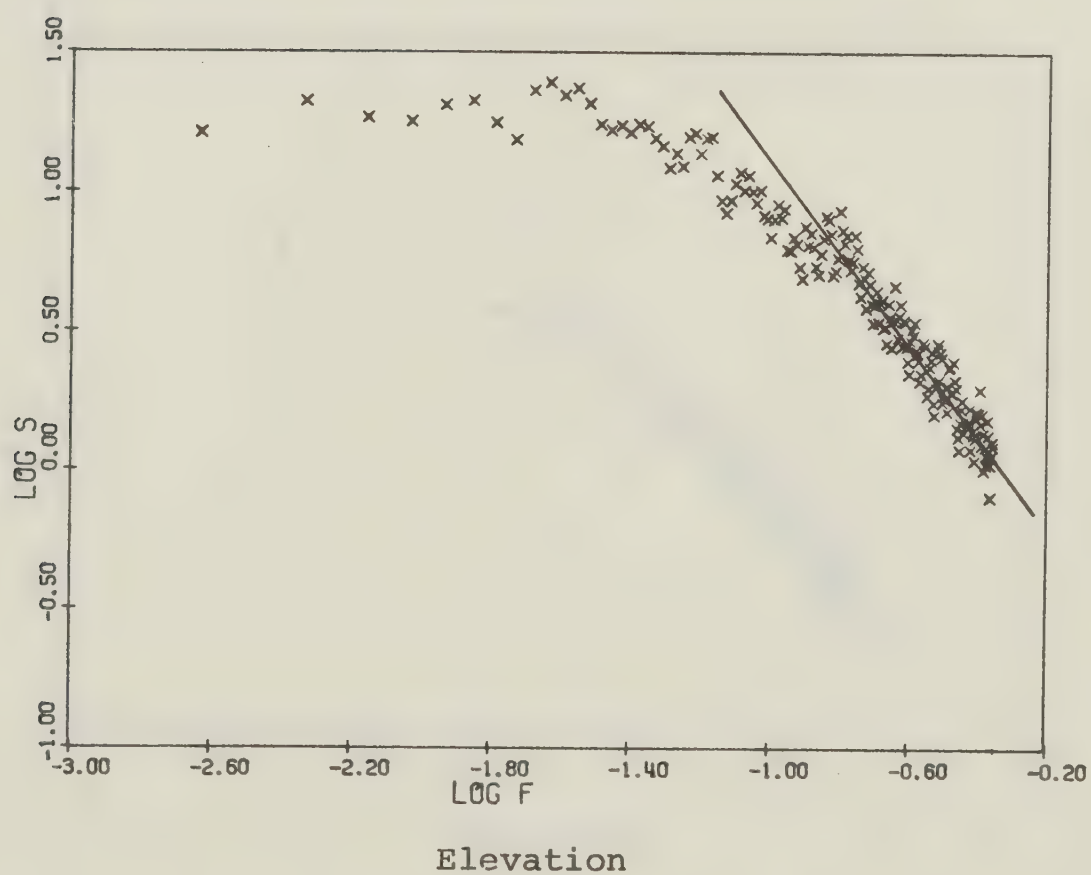
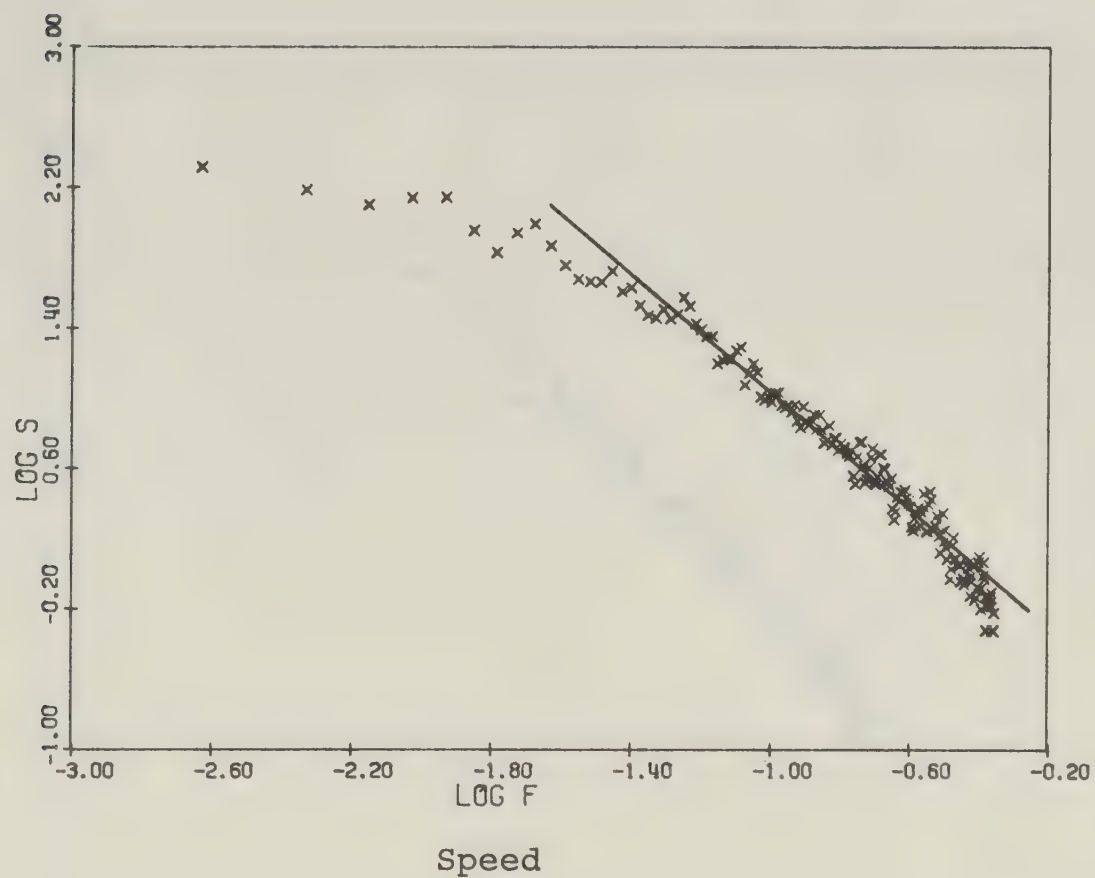


Figure A-14 Urban speed and elevation spectra, September 4
(log S vs log f)

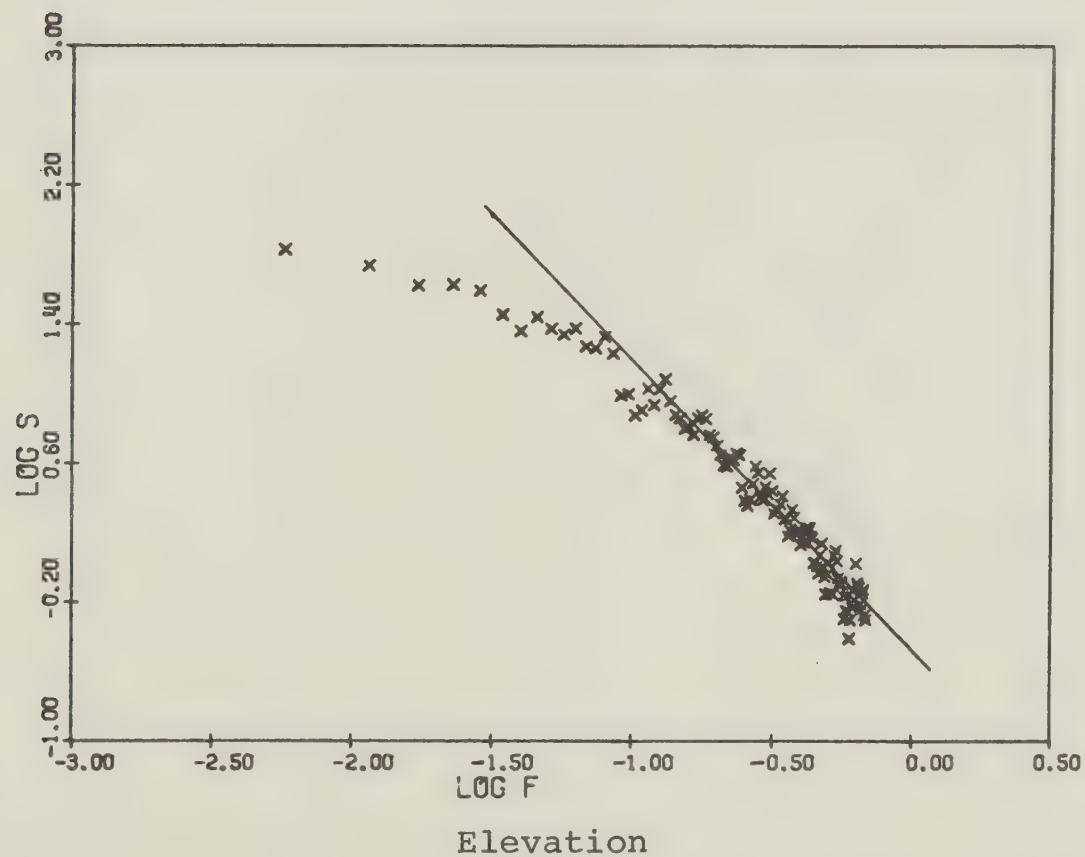
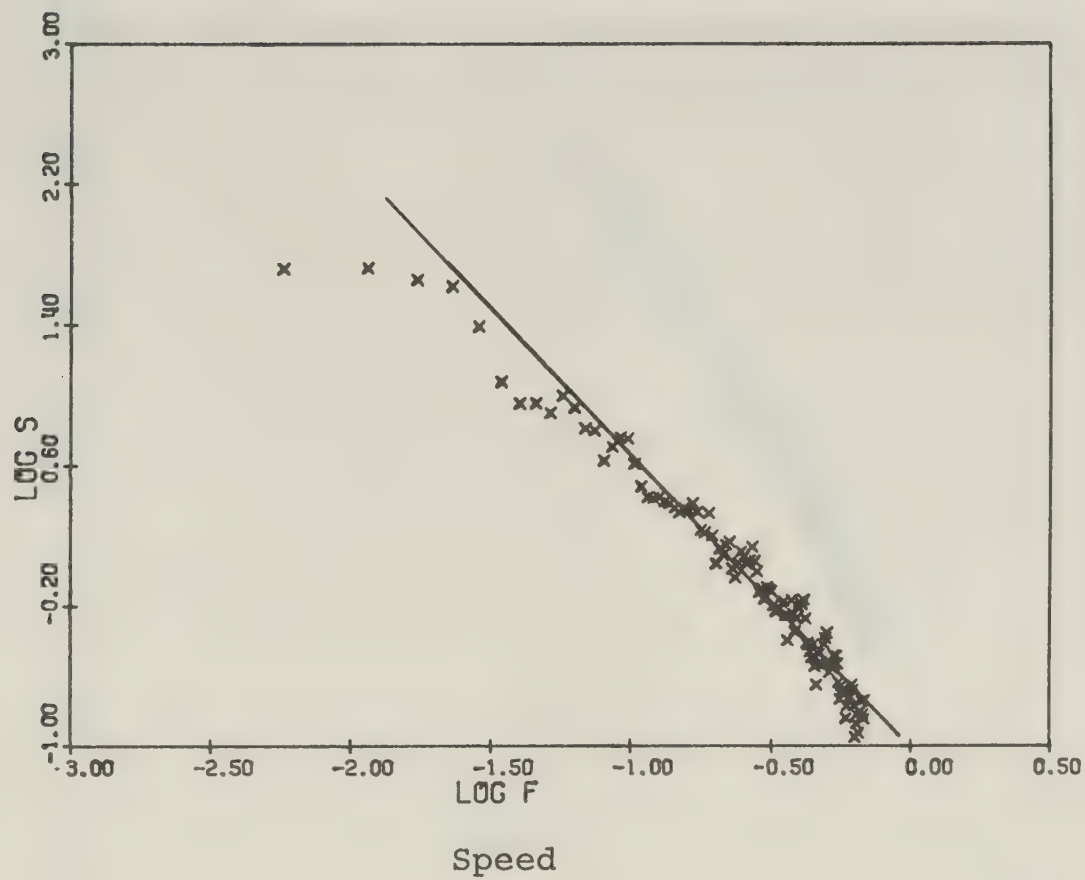


Figure A-15 Urban speed and elevation spectra, September 5
(log S vs log f)

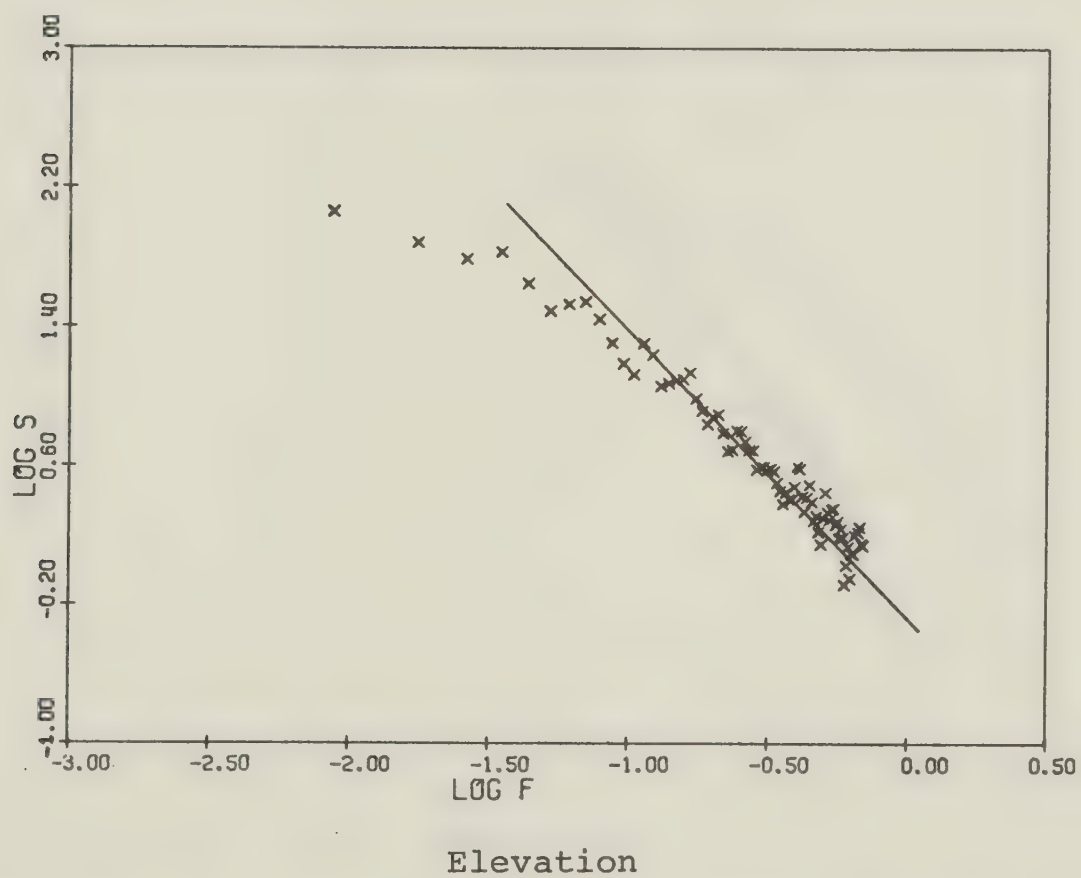
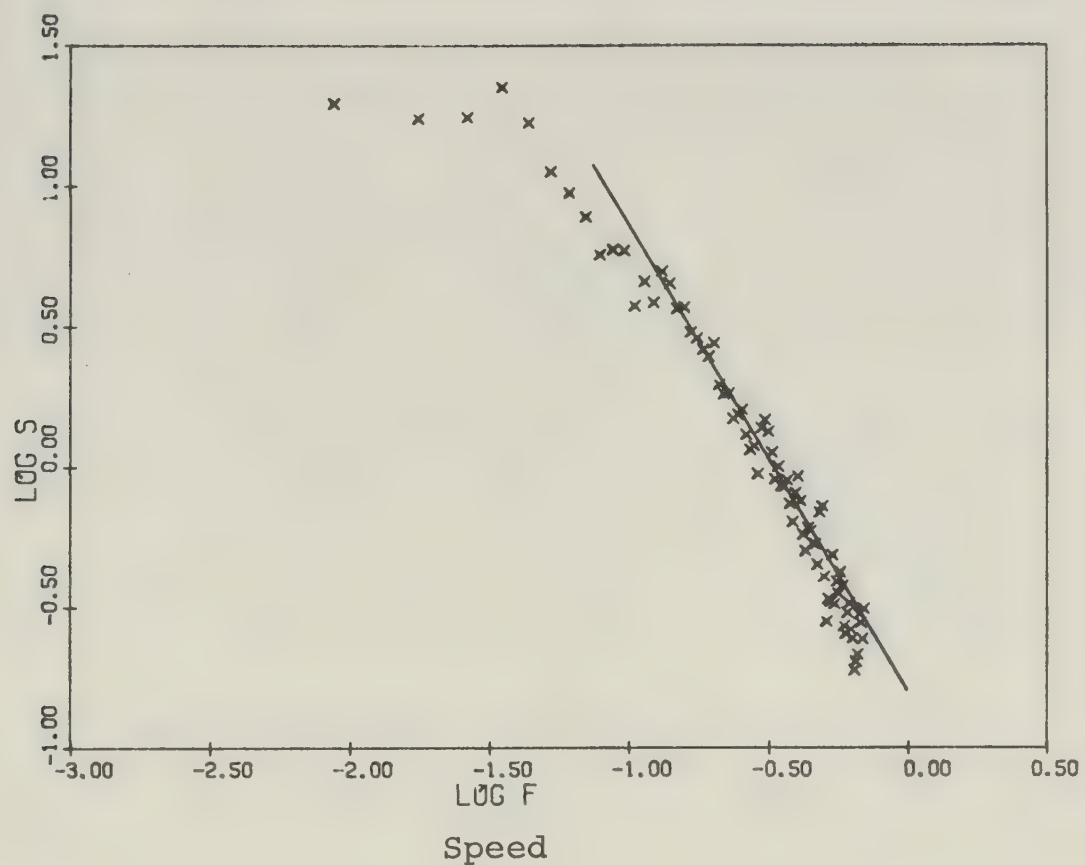


Figure A-16 Urban speed and elevation spectra, September 9
(log S vs log f)

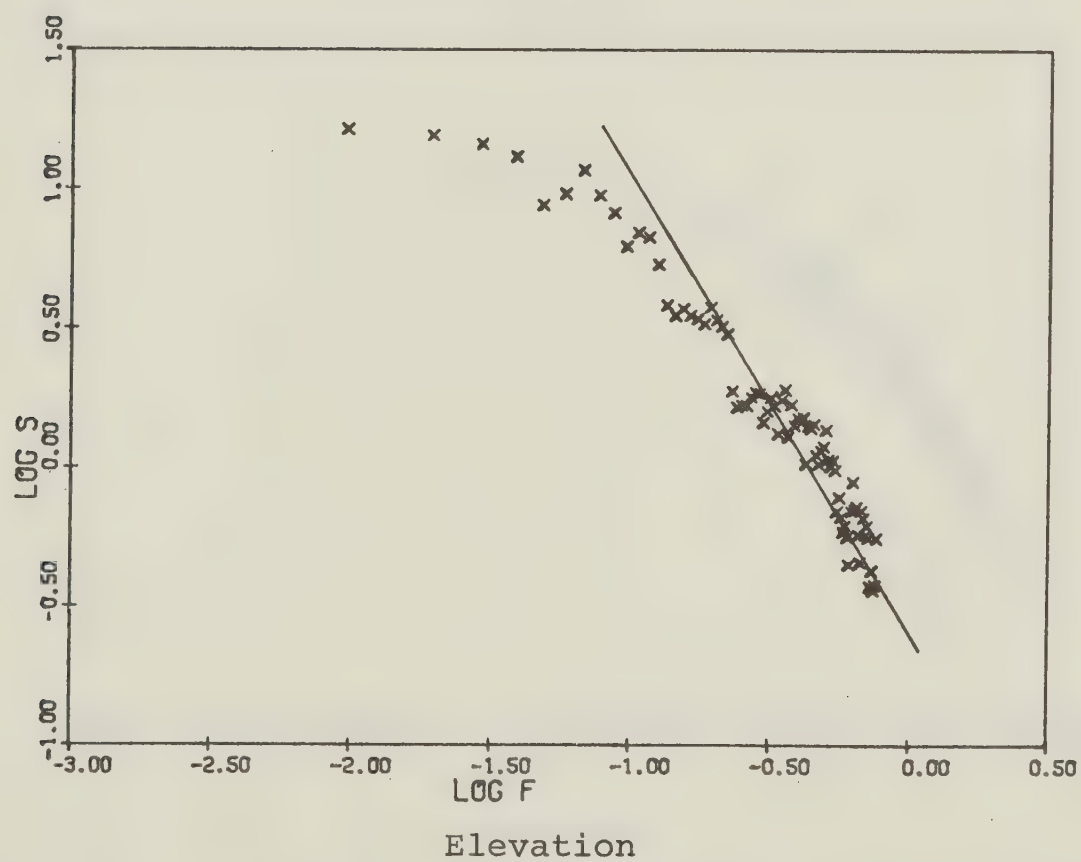
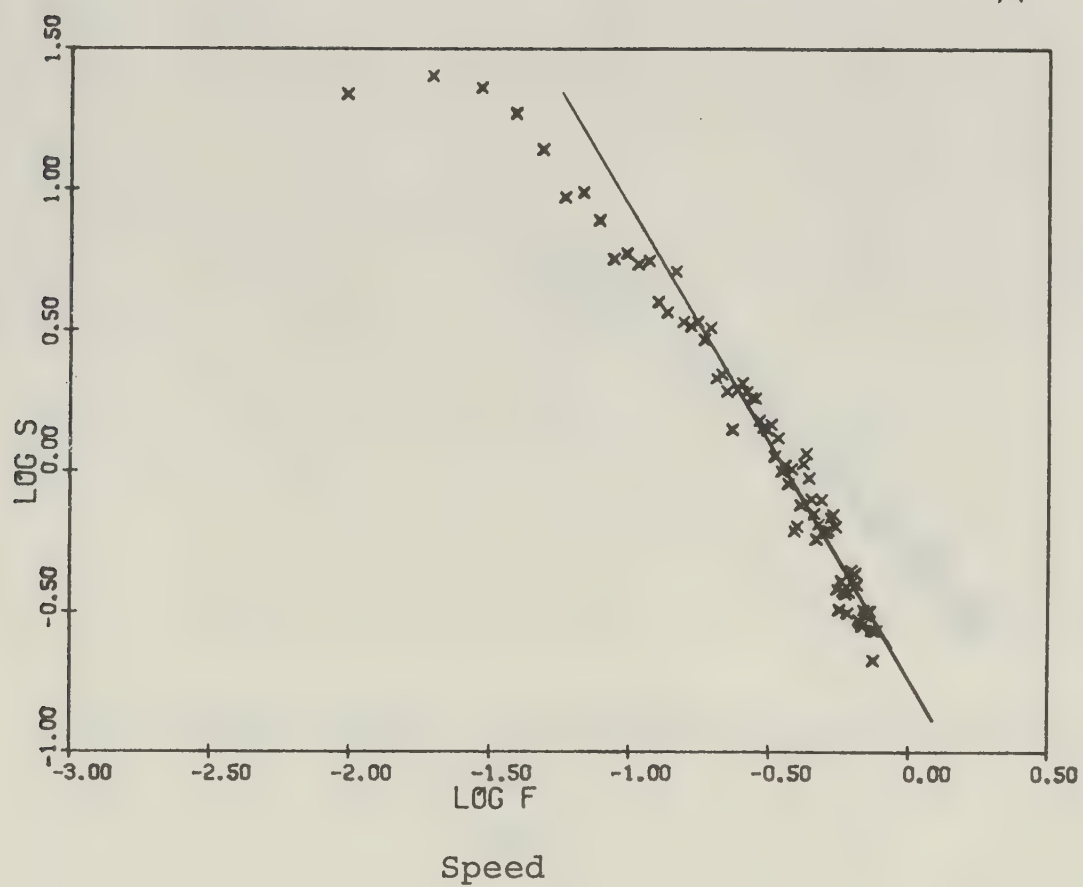


Figure A-17 Urban speed and elevation spectra, September 13
(log S vs log f)

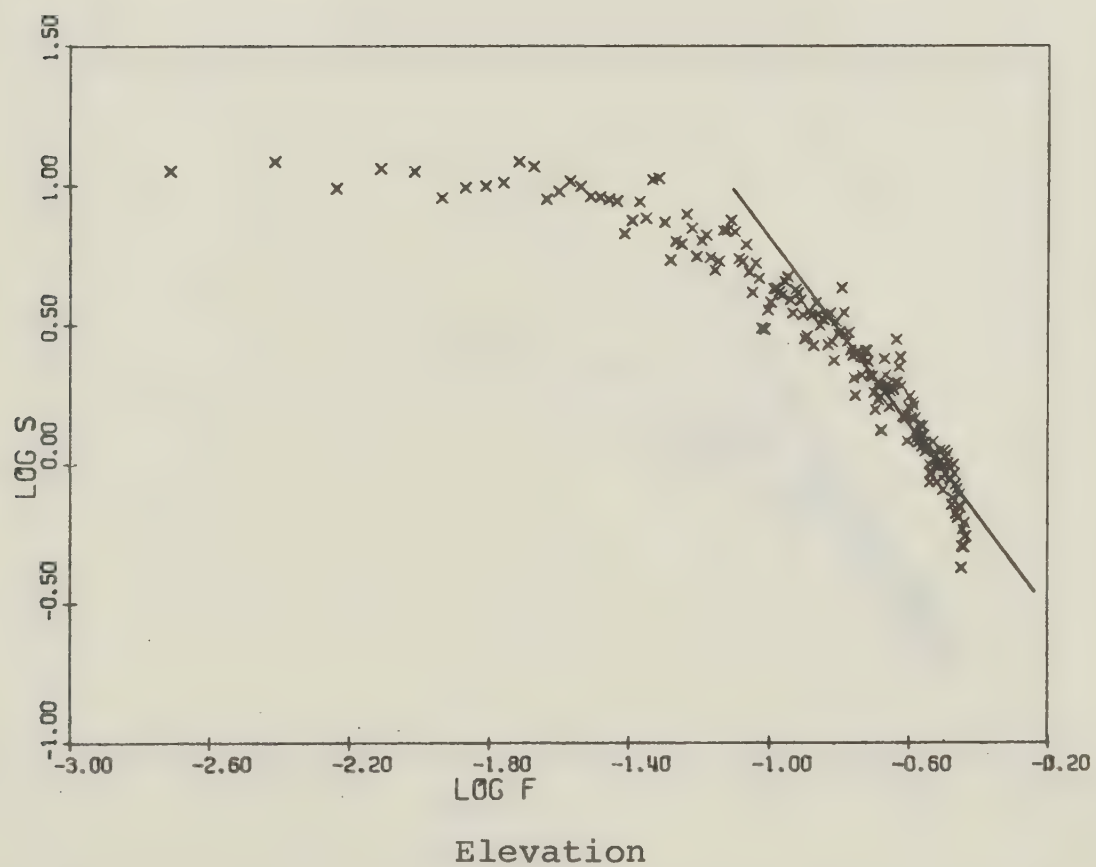
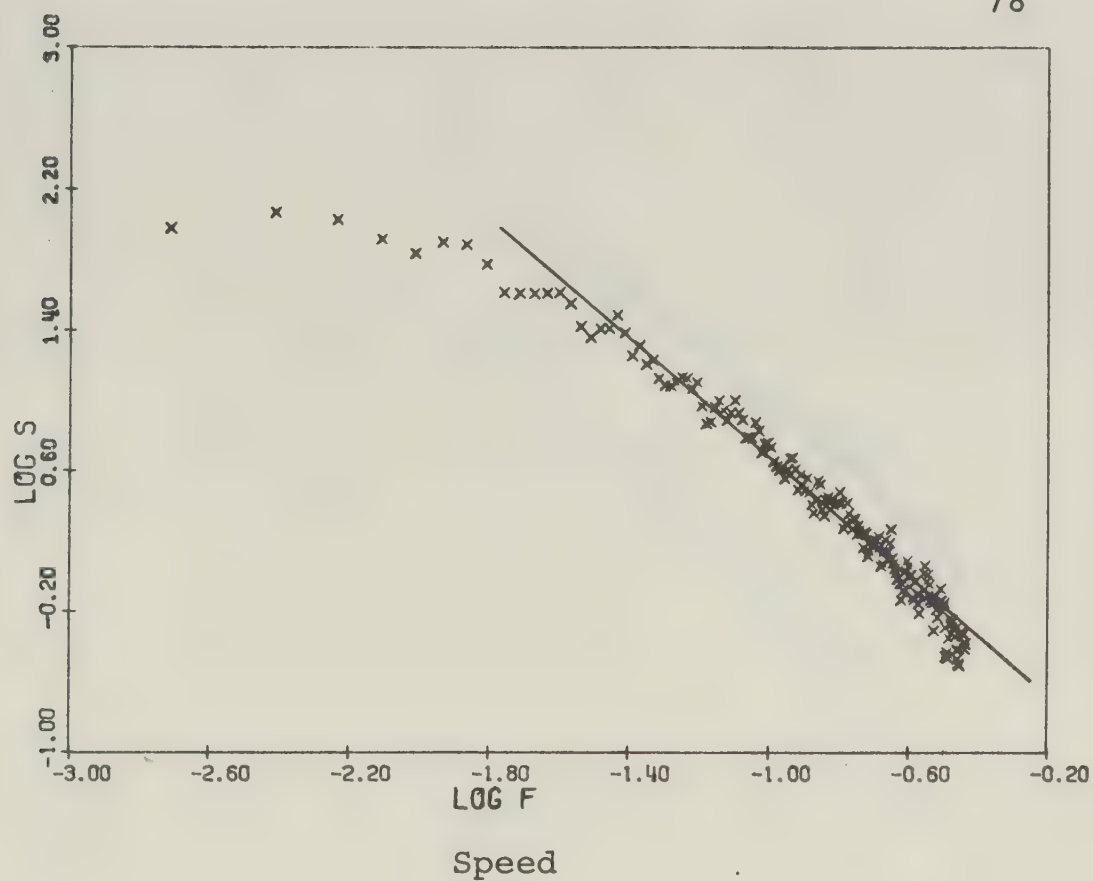


Figure A-18 Urban speed and elevation spectra, September 20
(log S vs log f)

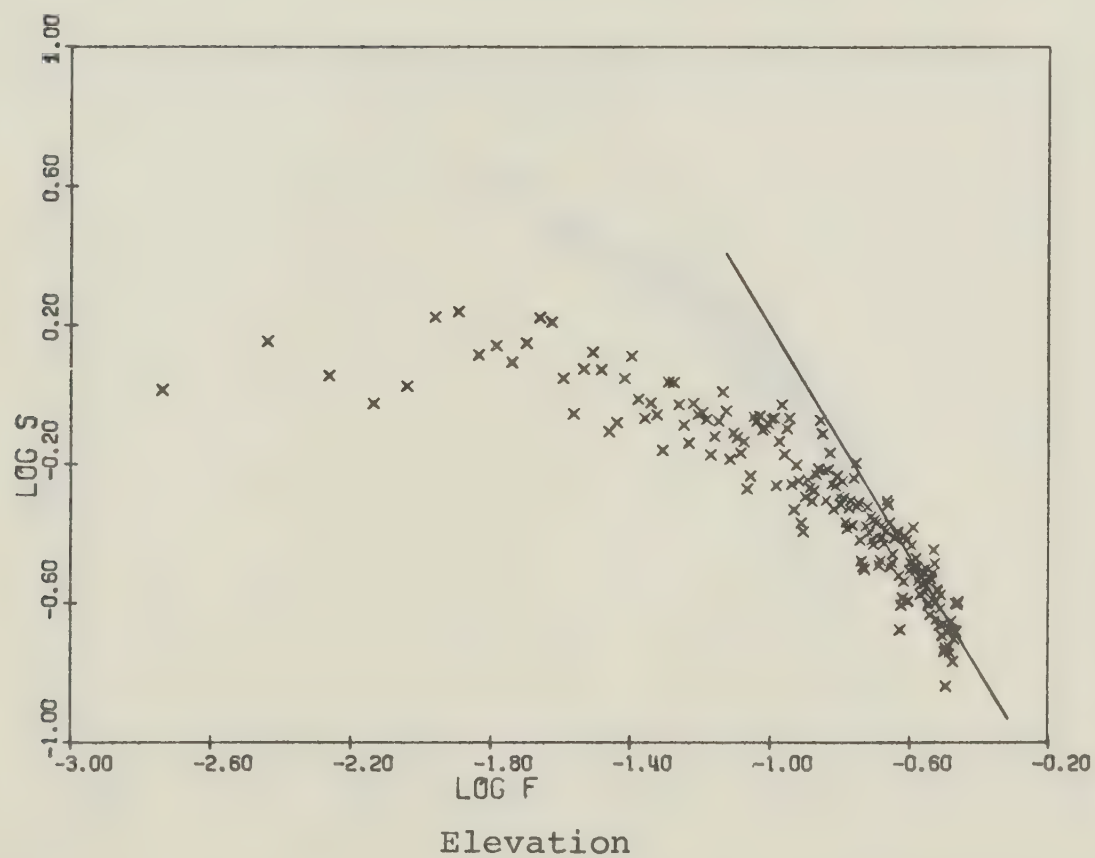
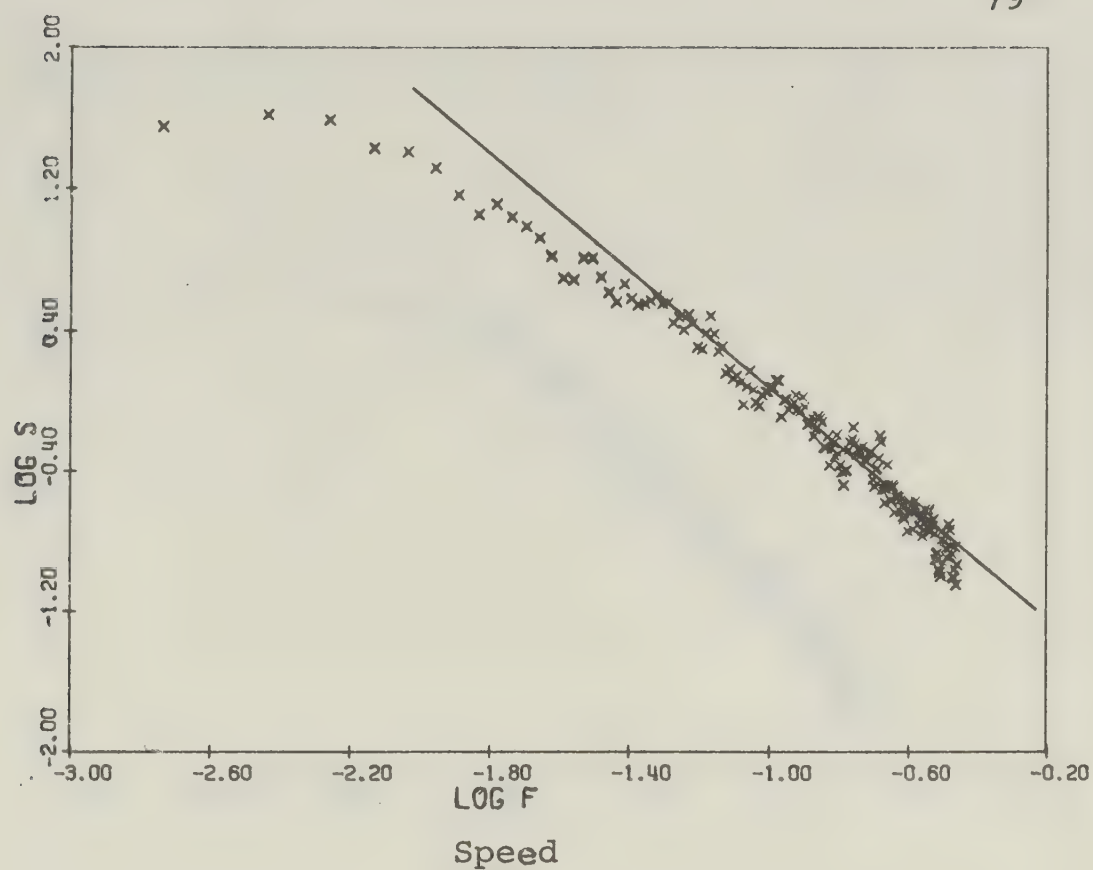


Figure A-19 Rural speed and elevation spectra, September 4
(log S vs log f)

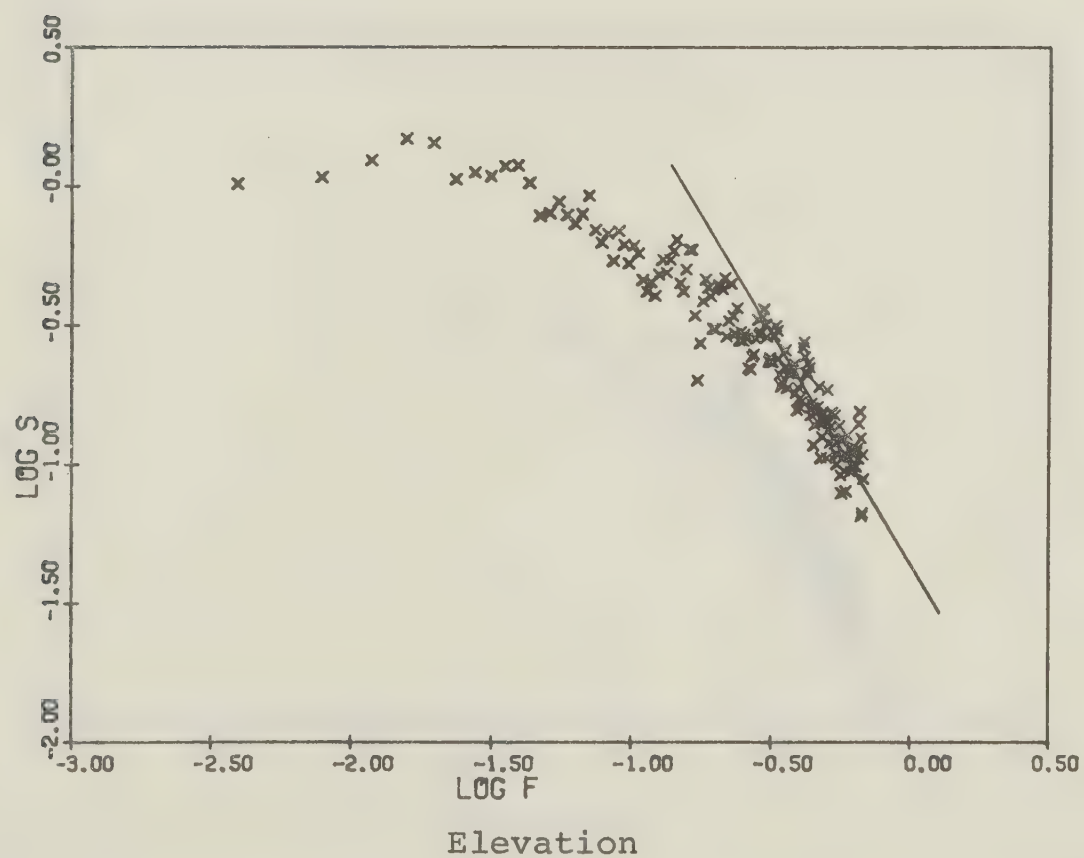
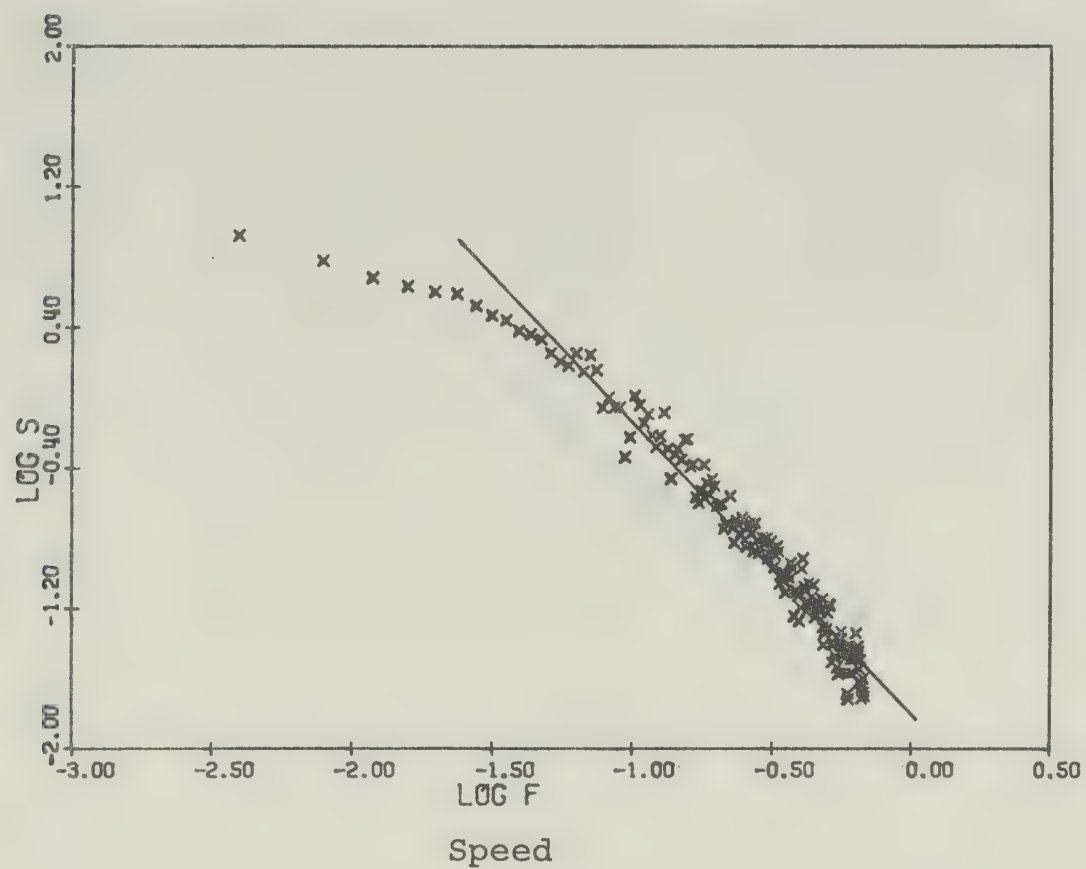


Figure A-20 Rural speed and elevation spectra, September 5
(log S vs log f)

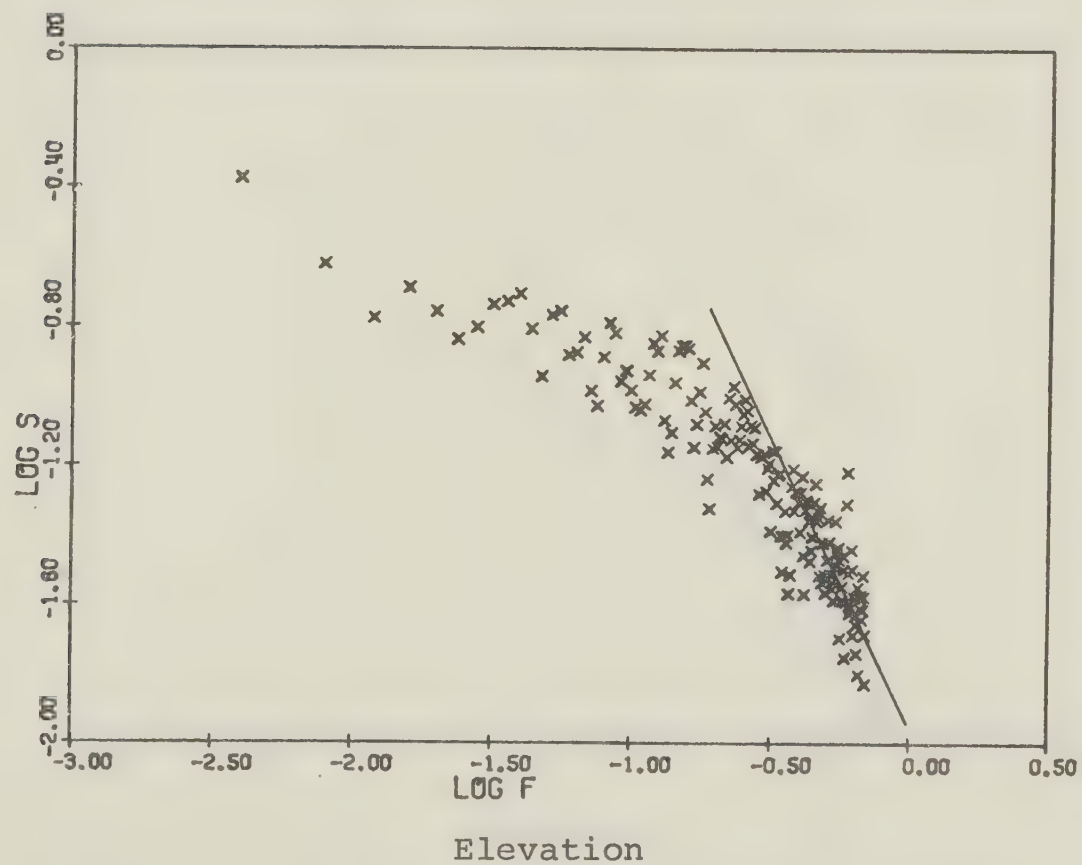
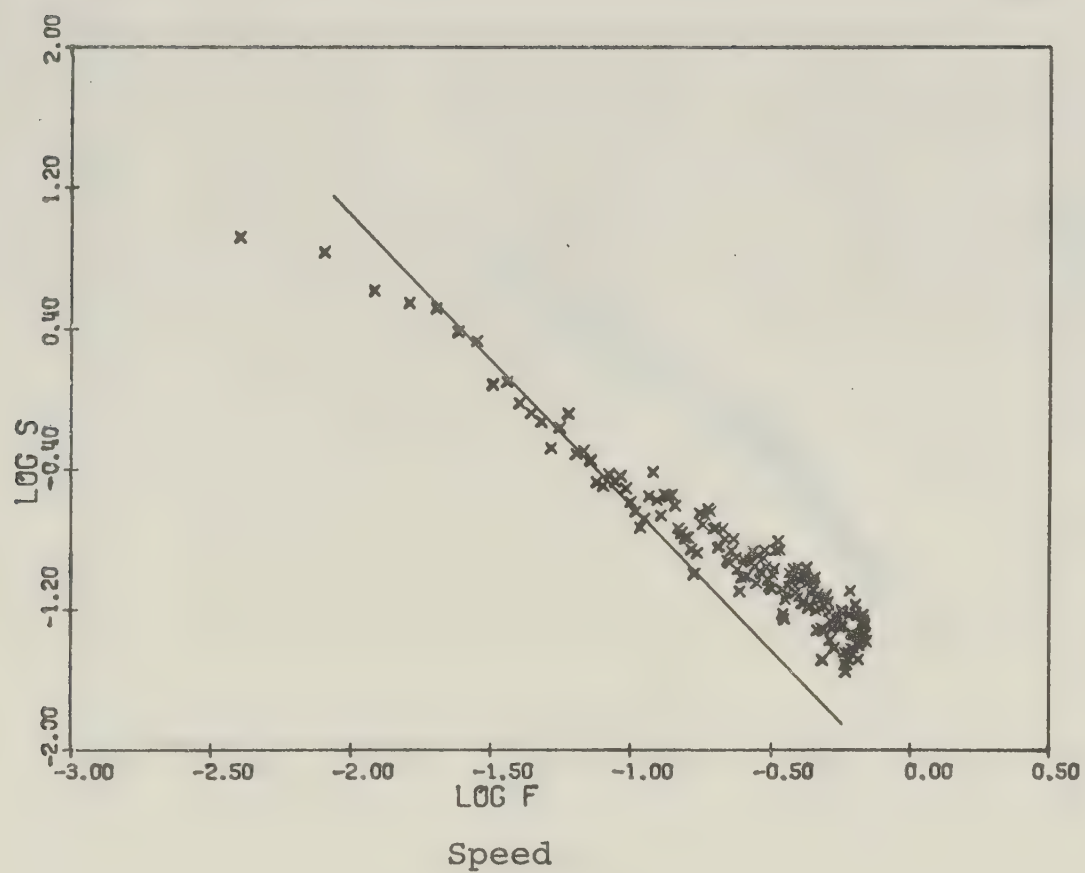


Figure A-21 Rural speed and elevation spectra, September 13
(log S vs log f)

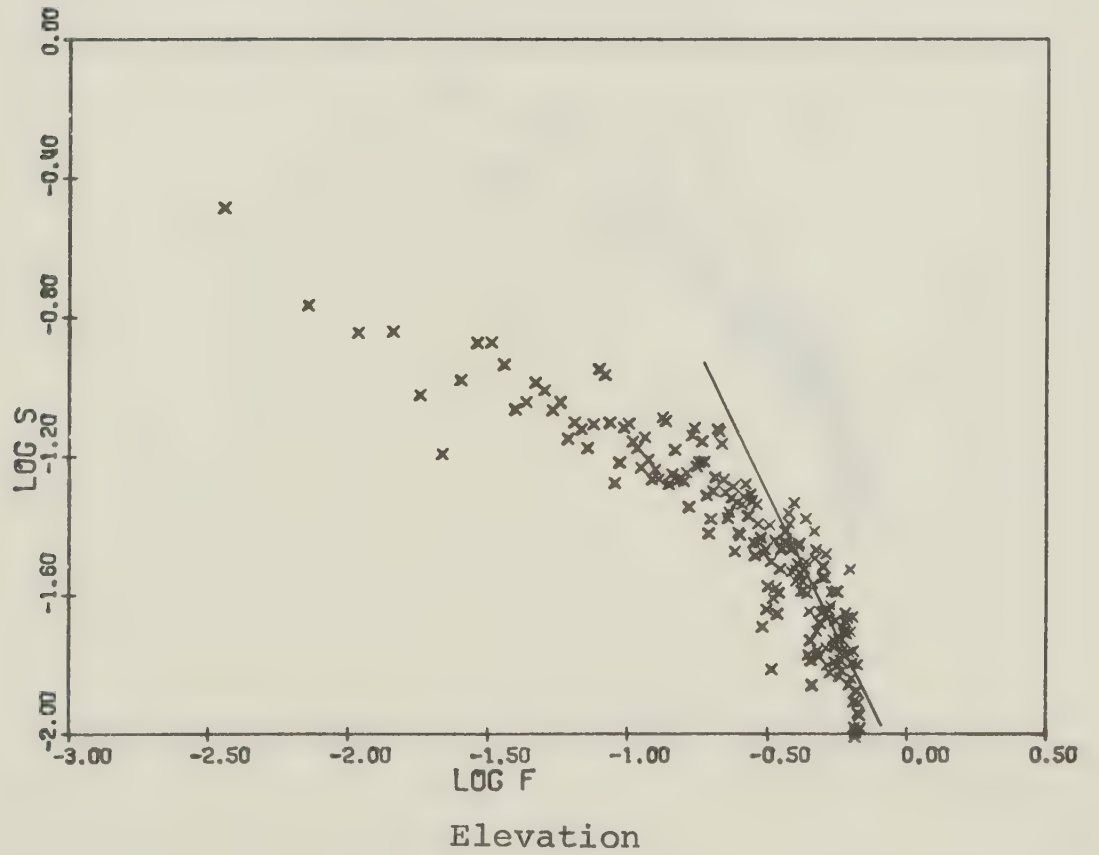
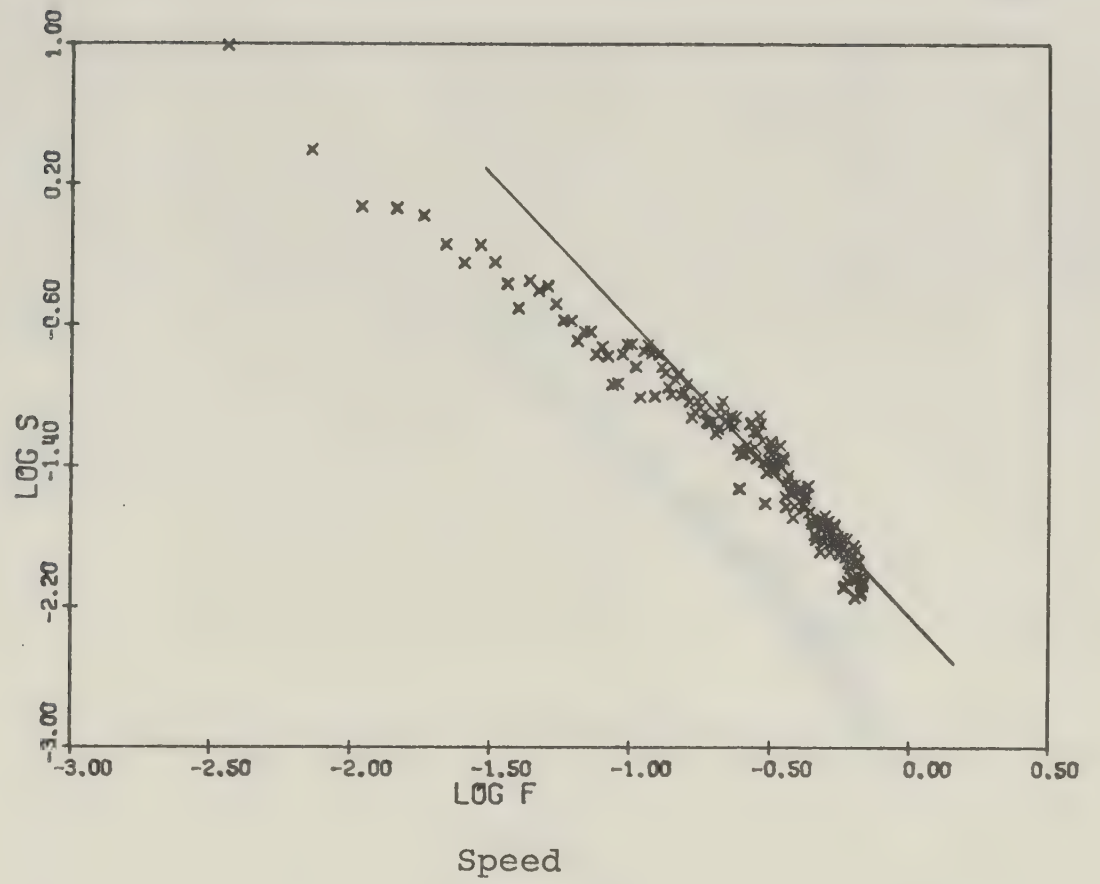


Figure A-22 Rural speed and elevation spectra, September 17
(log S vs log f)

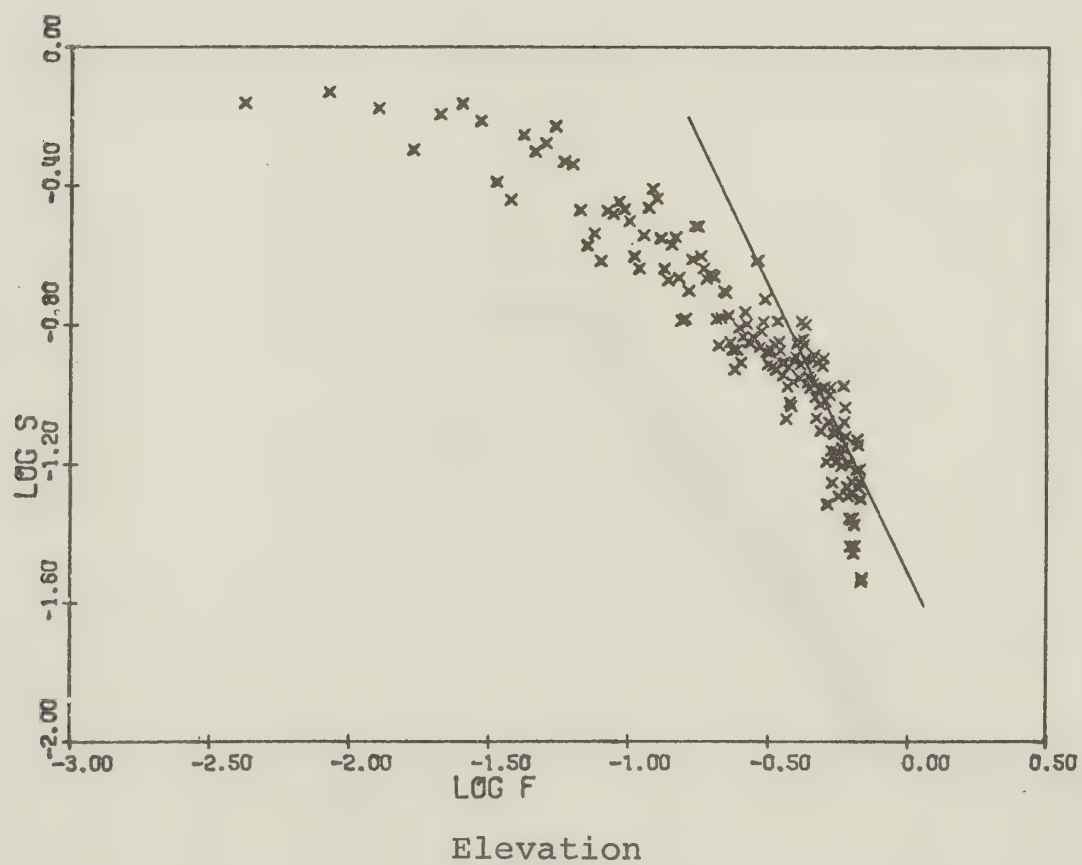
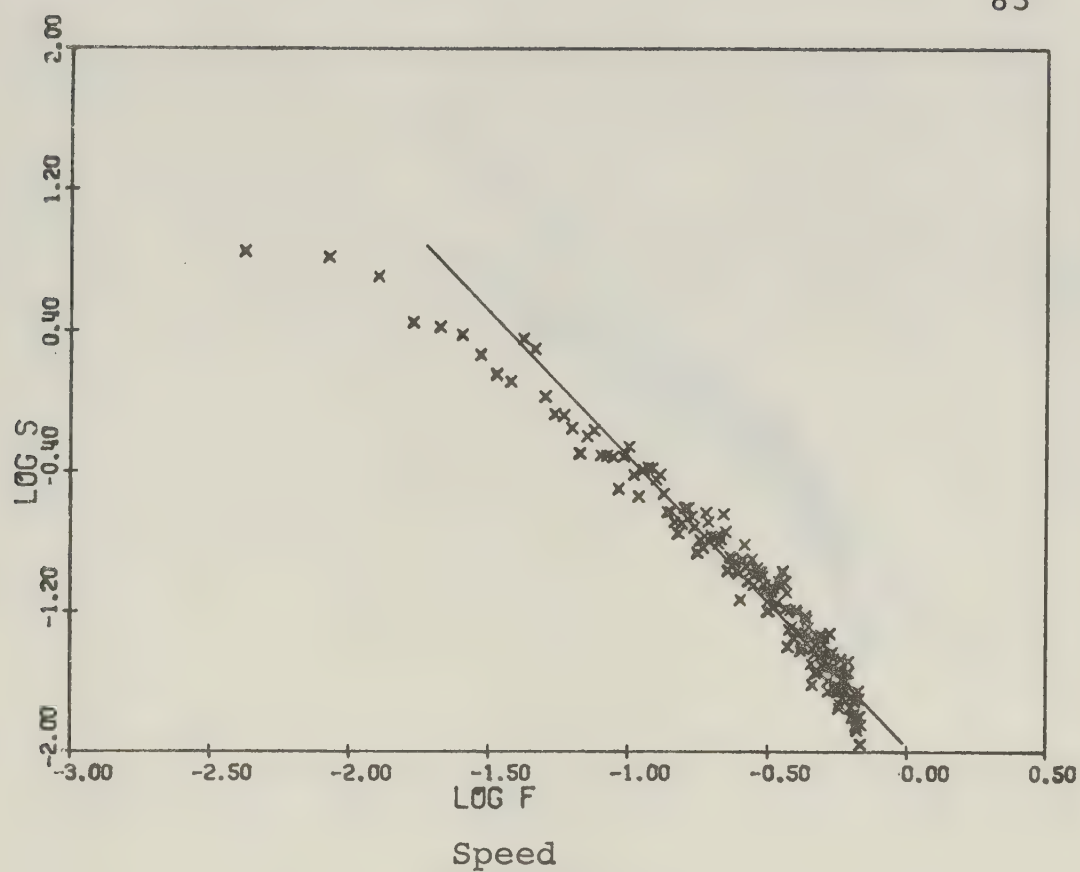
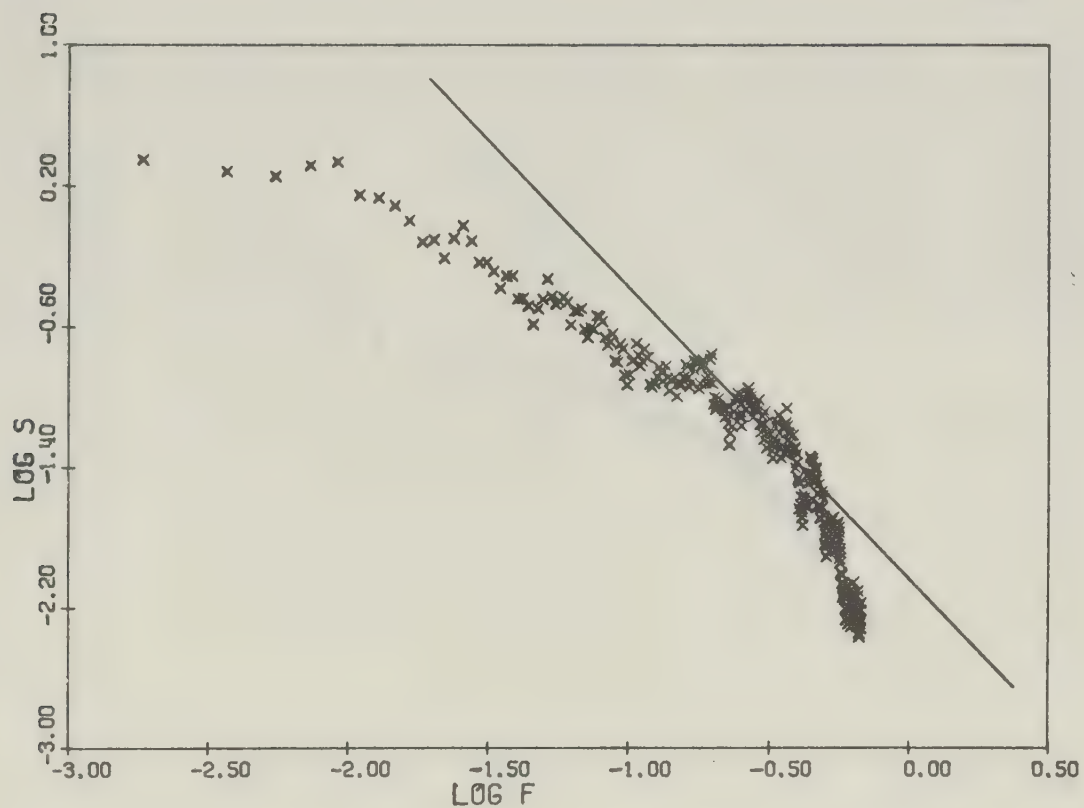
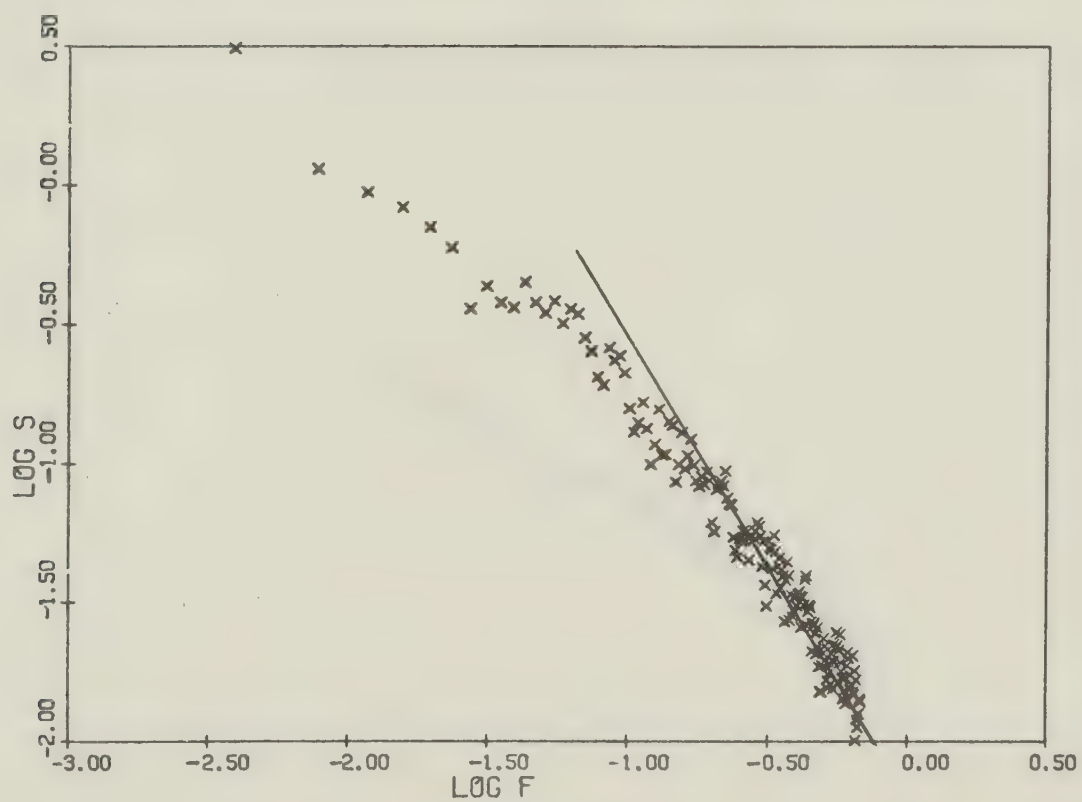


Figure A-23 Rural speed and elevation spectra, September 20
(log S vs log f)

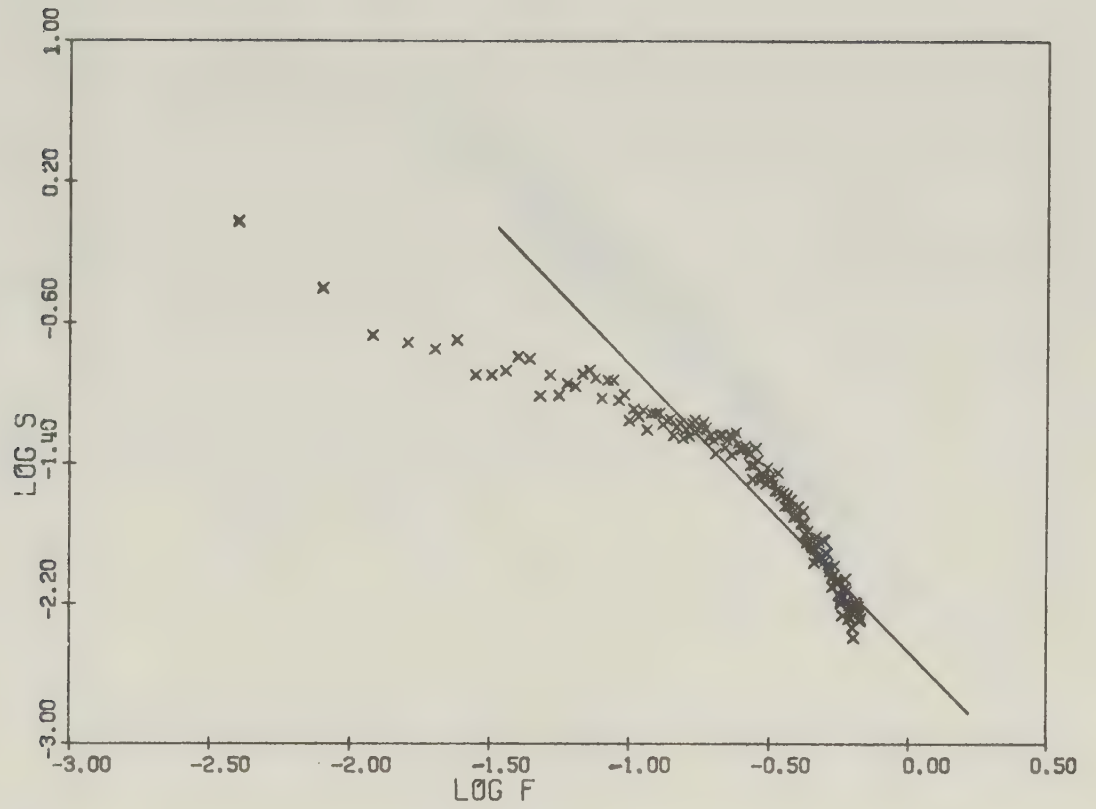


September 4

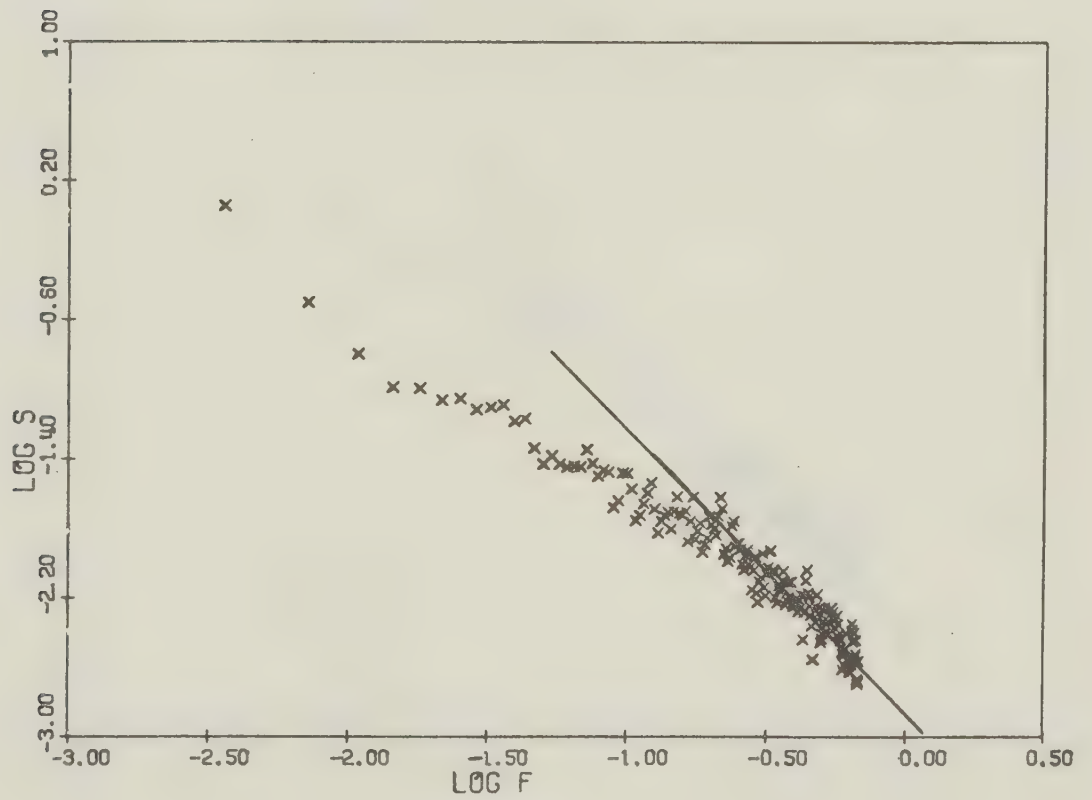


September 5

Figure A-24 Rural azimuth spectra, September 4 and 5
($\log S$ vs $\log f$)



September 13



September 17

Figure A-25 Rural azimuth spectra, September 13 and 17
(log S vs log f)

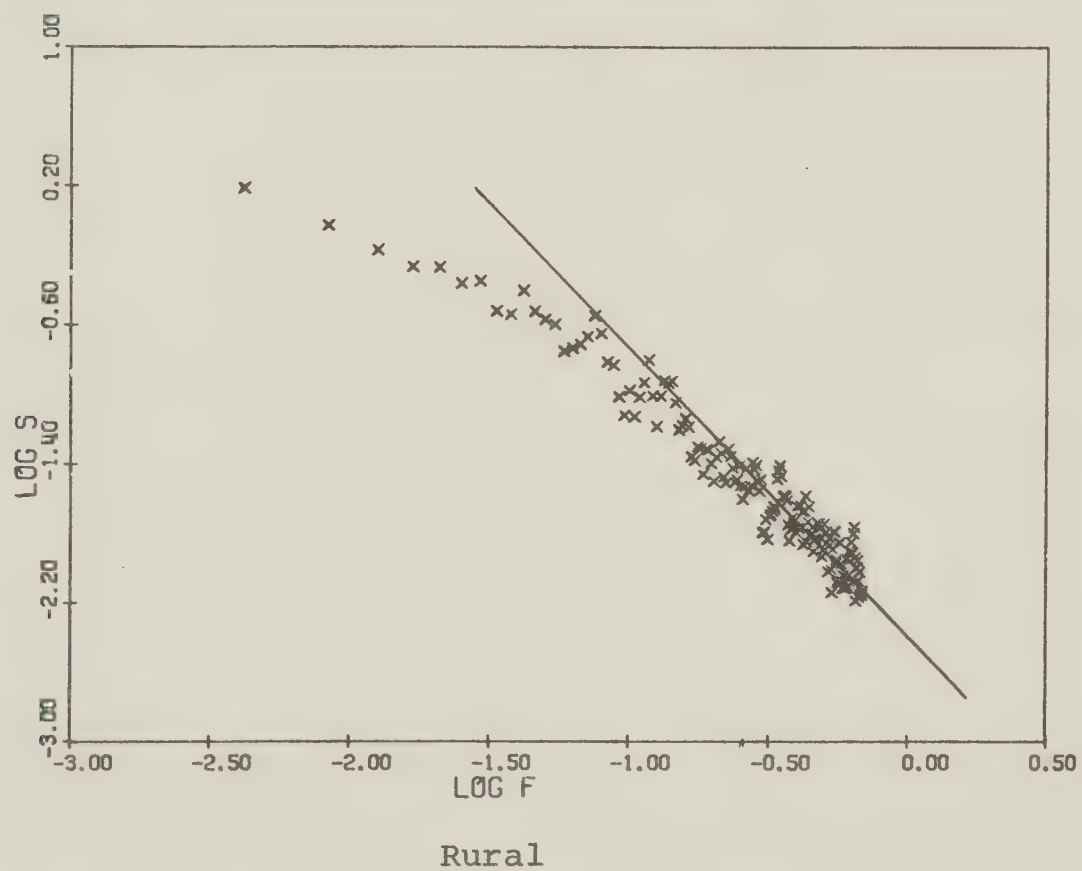
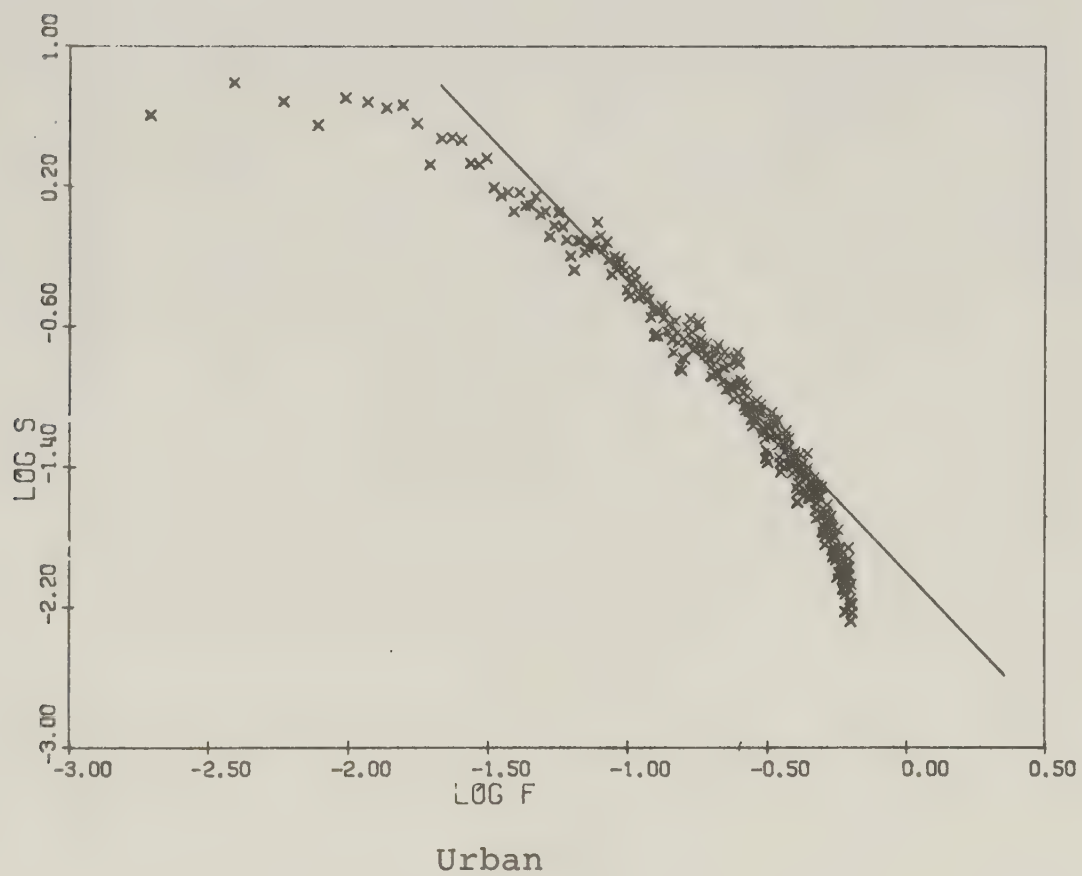


Figure A-26 Urban and rural azimuth spectra, September 20
(log S vs log f)

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